PREDICTIVE MODELS FOR WIRELESS FADING CHANNELS

BY

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ABSTRACT

This thesis presents methods for improving the performance of wireless networks through the modeling and prediction of time-varying multipath channels. The Rayleigh fading channel is characterized using first and second-order autoregressive (AR) time-series models. The AR processes model the channel variations at the time-scale of the characteristic Doppler frequency. Small time-scale variations are captured using linear interpolation of the AR model predictions. For defined error performance metrics, the fading signal is characterized using a state-space model that partitions the continuous amplitude variations into error and error-free states. The model parameters and state thresholds are derived as a function of Doppler frequency, signal-to-noise ratio and specified error probability. The aforementioned model is applied for estimating the probability of error as a function of transmission block size. The second-order AR model captures the pseudo-periodic behavior of the Rayleigh channel and produces accurate estimates of block error probabilities relative to the first-order AR process. The fading channel model is applied for both flat and frequency-selective channels in the design of a channel estimator and predictor. A Kalman-filter is designed to forecast the expected channel conditions and the predictions are applied for error control. The performance of the proposed error control approach is compared to a decision feedback equalizer (DFE). The model-based prediction shows at least a 25% improvement over the DFE.
ACKNOWLEDGEMENTS

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Poorna, Avichal, Anurag
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<th>Description</th>
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<tbody>
<tr>
<td>ACF</td>
<td>Autocorrelation Function</td>
</tr>
<tr>
<td>AR</td>
<td>Autoregressive Process</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>BER</td>
<td>Bit Error Rate</td>
</tr>
<tr>
<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
</tr>
<tr>
<td>BS</td>
<td>Base Station</td>
</tr>
<tr>
<td>CDF</td>
<td>Cumulative Density Function</td>
</tr>
<tr>
<td>CIR</td>
<td>Channel Impulse Response</td>
</tr>
<tr>
<td>dB</td>
<td>Decibels</td>
</tr>
<tr>
<td>DFE</td>
<td>Decision Feedback Equalizer</td>
</tr>
<tr>
<td>FIR</td>
<td>Finite Impulse Response</td>
</tr>
<tr>
<td>ISI</td>
<td>Inter Symbol Interference</td>
</tr>
<tr>
<td>LOS</td>
<td>Line Of Sight</td>
</tr>
<tr>
<td>LMS</td>
<td>Least Mean Square</td>
</tr>
<tr>
<td>MC</td>
<td>Markov Chain</td>
</tr>
<tr>
<td>MS</td>
<td>Mobile Station</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Definition</td>
</tr>
<tr>
<td>--------------</td>
<td>------------</td>
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<tr>
<td>MSE</td>
<td>Mean Square Error</td>
</tr>
<tr>
<td>NLOS</td>
<td>Non-Line of Sight</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>PDP</td>
<td>Power Delay Profile</td>
</tr>
<tr>
<td>RLS</td>
<td>Recursive Least Square</td>
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<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
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<tr>
<td>T-R</td>
<td>Transmitter-Receiver pair</td>
</tr>
<tr>
<td>WSS</td>
<td>Wide Sense Stationary</td>
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**LIST OF SYMBOLS**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$s(t)$</td>
<td>Transmitted low-pass signal</td>
</tr>
<tr>
<td>$a(t)$</td>
<td>Amplitude of transmitted low-pass signal</td>
</tr>
<tr>
<td>$\theta(t)$</td>
<td>Phase of transmitted low-pass signal</td>
</tr>
<tr>
<td>$f_c$</td>
<td>Carrier frequency</td>
</tr>
<tr>
<td>$s_b(t)$</td>
<td>Transmitted band-pass signal</td>
</tr>
<tr>
<td>$r_b(t)$</td>
<td>Received band-pass signal</td>
</tr>
<tr>
<td>$a_n(t)$</td>
<td>Amplitude of $n^{th}$ multipath component</td>
</tr>
<tr>
<td>$\phi_n(t)$</td>
<td>Phase of $n^{th}$ multipath component</td>
</tr>
<tr>
<td>$\tau_n(t)$</td>
<td>Delay of $n^{th}$ multipath component</td>
</tr>
<tr>
<td>$D_n(t)$</td>
<td>Doppler frequency shift of $n^{th}$ multipath component</td>
</tr>
<tr>
<td>$v(t)$</td>
<td>Mobile speed</td>
</tr>
<tr>
<td>$\beta_n(t)$</td>
<td>Angle between mobile velocity vector and ray incident on mobile</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Path-loss exponent</td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>Wave-length of carrier wave</td>
</tr>
<tr>
<td>$f_m(t)$</td>
<td>Doppler frequency</td>
</tr>
<tr>
<td>$r(t)$</td>
<td>Received low-pass signal</td>
</tr>
</tbody>
</table>
\( h(\tau; t) \) Channel impulse response
\( \Delta f_c \) Coherence bandwidth
\( \Delta t_c \) Coherence time
\( \Delta f_s \) Transmission bandwidth
\( \Delta t_s \) Symbol interval
\( B_d \) Doppler spread
\( E[. ] \) Expected value of the argument
\( Re[. ] \) Real component of the complex argument
\( Im[. ] \) Imaginary component of the complex argument
\( Var(.) \) Variance of the argument
\( Pr[. ] \) Probability of the argument
\( \Delta t \) Autocorrelation lag
\( \rho_h(\Delta t) \) Autocorrelation of channel impulse response at lag \( \Delta t \)
\( \bar{\tau} \) Mean delay
\( \sigma^2_\tau \) Channel delay spread
\( H(f; t) \) Channel impulse response in frequency domain
\( \rho_H(\Delta t) \) Autocorrelation of frequency domain channel impulse response
\( S_H(\Delta f; \lambda) \) Spectral density of frequency domain channel impulse response
\( y(t) \) Inphase component of received low-pass signal
$y_k$  Inphase component at instant $k$, discrete value of $y(t)$

$z(t)$  Quadrature component of low-pass received signal

$z_k$  Quadrature component at instant $k$, discrete value of $z(t)$

$x(t)$  Magnitude of inphase $y(t)$ and quadrature $z(t)$ components

$x_k$  Magnitude at instant $k$, discrete value of $x(t)$

$x_{\text{mean}}$  Mean of Rayleigh distribution

$f_{Y}(y)$  PDF of inphase component $y(t)$, Gaussian distributed

$f_{Z}(z)$  PDF of quadrature component $z(t)$, Gaussian distributed

$f_{X}(x)$  PDF of magnitude $x(t)$; Rayleigh distributed

$\sigma_{Y}^{2}$  Variance of inphase component

$\sigma_{Z}^{2}$  Variance of quadrature component

$\sigma_{X}^{2}$  Variance of magnitude

$\rho_{rr'\Delta t}$  Autocorrelation coefficient of received band-pass signal at lag $\Delta t$

$\rho_{yy}(\Delta t)$  Autocorrelation coefficient of the inphase component at lag $\Delta t$

$\rho_{zz}(\Delta t)$  Autocorrelation coefficient of the quadrature component at lag $\Delta t$

$\rho_{yz}(\Delta t)$  Cross correlation coefficient of inphase/quadrature components at $\Delta t$

$\frac{\Omega_{p}}{2}$  Average power in inphase/quadrature multipath components

$J_{0}(.)$  Bessel function of first kind, zeroth order

$S_{Y}(f)$  Power spectral density of inphase component
$S_Z(f)$  Power spectral density of quadrature component

$N(\mu, \sigma^2)$  Gaussian distribution, with mean $\mu$ and variance $\sigma^2$

$N_R$  Average level crossing rate at level $R$

$\bar{T}$  Average fade duration

$\eta(t)$  Additive white Gaussian noise (AWGN)

$S_{\eta}(f)$  Power spectral density of AWGN in channel

$\frac{N_0}{2}$  Variance of channel noise

$f_N(\eta)$  PDF of AWGN noise

$\varepsilon_b$  Power of transmitted signal (per bit)

$f_R(r)$  PDF of received low-pass signal

$D_{-2}(.)$  Parabolic cylinder function

$\Phi(.)$  Error function

$P_e$  Average probability of bit errors

$Q(.)$  Complementary error function

$\bar{\gamma}_b$  Average received SNR over Rayleigh fading channels

$\gamma_b$  Instantaneous received SNR over Rayleigh fading channels

$f_{\gamma_b}(\gamma_b)$  PDF of instantaneous received SNR

$\chi_{TH}$  Error threshold on fading channels

$C$  Fraction of total error caused at error threshold $\chi_{TH}$
\( s_i \)  
State of Markov chain

\( S_n \)  
State assumed by MC at instant \( n \)

\( P \)  
Transition probability matrix for Markov chain model

\( p_{ij} \)  
Probability of transition from state \( s_i \) to \( s_j \)

\( \pi_i \)  
Steady-state probability in state \( i \)

\( \Pi \)  
Steady-state probability vector

\( s_{\text{mean}} \)  
Mean of Markov chain

\( \Lambda \)  
Covariance matrix of inphase/quadrature component

\( \lambda_{mn} \)  
Element of covariance matrix \( \Lambda \)

\( I_0(\cdot) \)  
Modified Bessel function of zeroth order

\( \rho_k \)  
ACF \( \rho(k\Delta t) \)

\( \Phi_i \)  
\( i^{\text{th}} \) autoregressive coefficient

\( \sigma_N^2 \)  
Variance of AR noise process

\( \hat{y}^{(1)}_n \)  
Forecast of inphase component using first order memory

\( \hat{y}^{(2)}_n \)  
Forecast of inphase component using second order memory

\( s_g \)  
Good channel state, possible values: \( s_1, s_2 \)

\( s_b \)  
Bad channel state, possible value: \( s_2 \)

\( \mu^{(1)} \)  
Conditional expectation using first order memory

\( \sigma^{(1)2} \)  
Conditional variance using first order memory
\( \mu^{(2)} \)  
Conditional expectation using second order memory

\( \sigma^{(2)2} \)  
Conditional variance using second order memory

\( L \)  
Sampling lag

\( L_g \)  
Sampling lag when channel transition originates from good state

\( L_{g\rightarrow b} \)  
Sampling lag when channel transitions from good to bad state

\( L_{g\rightarrow g} \)  
Sampling lag when channel transitions from good to good state

\( L_b \)  
Sampling lag when channel transition originates from bad state

\( L_{b\rightarrow b} \)  
Sampling lag when channel transitions from bad to bad state

\( L_{b\rightarrow g} \)  
Sampling lag when channel transitions from bad to good state

\( L_{\text{max}} \)  
Maximum value of sampling lag

\( L_{gg} \)  
Sampling lag when two good states are considered

\( L_{gg\rightarrow b} \)  
Sampling lag when channel transitions from good to bad state

\( L_{gg\rightarrow g} \)  
Sampling lag when channel transitions from good to good state

\( L_{bb} \)  
Sampling lag when two bad states are considered

\( L_{bb\rightarrow g} \)  
Sampling lag when channel transitions from bad to good state

\( L_{bb\rightarrow b} \)  
Sampling lag when channel transitions from bad to bad state

\( F \)  
Forecast step in multiples of sampling lag

\( e^2_n(F) \)  
Mean square error for forecast at \( F \)

\( B \)  
Backward shift operator
\( P(m, n) \)  Probability of \( m \) errors in a block size of \( n \) bits

\( P_i(m, n) \)  Probability of \( m \) errors in a block size of \( n \) bits when start state is \( s_i \)

\( D \)  Interleaving delay

\( H(z) \)  CIR in \( z \)-domain

\( R(z) \)  Received signal in \( z \)-domain

\( S(z) \)  Transmitted signal in \( z \)-domain

\( \alpha_i, \beta_i \)  Coefficients of Pole-zero model

\( h_k \)  \( k^{th} \) CIR tap

\( h_k^i \)  \( k^{th} \) CIR tap at instant \( i \)

\( \bar{h}_i \)  Mean value of \( i^{th} \) CIR tap

\( g_k^i \)  Inphase component of CIR \( h_k^i \)

\( f_k^i \)  Quadrature component of CIR \( h_k^i \)

\( \sigma^2_{g_i} \)  Variance of \( g_k^i \)

\( \sigma^2_{f_i} \)  Variance of \( f_k^i \)

\( \zeta_k \)  Input signal at instant \( k \), discrete value of \( s(t) \)

\( \hat{\zeta}_k \)  Estimated symbol at \( k^{th} \) instant

\( p_k \)  Inphase component of signal \( \zeta_k \)

\( q_k \)  Quadrature component of signal \( \zeta_k \)

\( r_k \)  Received symbol at instant \( n \), discrete value of \( r(t) \)
\( u_k \) \hspace{1em} \text{Inphase component of signal } r_k \\
\( v_k \) \hspace{1em} \text{Quadrature component of signal } r_k \\
\( \eta_k \) \hspace{1em} \text{Noise at instant } n, \text{ discrete value of } \eta(t) \\
\( t_k \) \hspace{1em} \text{Inphase component of noise } \eta_k \\
\( \mu_k \) \hspace{1em} \text{Quadrature component of noise } \eta_k \\
\( L_k \) \hspace{1em} \text{Matrix of input symbols at instant } k \\
\( G_k \) \hspace{1em} \text{Matrix of inphase/quadrature component of CIR at instant } k \\
\( T \) \hspace{1em} \text{Transition matrix of inphase/quadrature CIR components} \\
\( E_k \) \hspace{1em} \text{Matrix of error terms for AR model for multipath CIR} \\
\( Q \) \hspace{1em} \text{Covariance matrix for error } E_k \\
\( R \) \hspace{1em} \text{Covariance matrix of estimation error} \\
\( c_k \) \hspace{1em} \text{Feedforward coefficients of Decision feedback equalizer} \\
\( b_k \) \hspace{1em} \text{Feedback coefficients of Decision feedback equalizer}
CHAPTER 1

INTRODUCTION

1.1 Features of Wireless Channels

A typical wireless transmission system is comprised of base stations (BS) and mobile stations (MS). The BS transmits and receives the wireless signal to and from the MS. Depending on the coverage area, the wireless systems can be classified into femtocells, picocells, microcells, macrocells and megacells [1]. Femtocells have the smallest coverage area ranging a few meters. They are used in personal computing environments where line of sight (LOS) communication is present. The picocells are typically deployed in indoor environments to support wireless local area networks. The communication is typically LOS, but the presence of obstructions between BS and MS may also result in a non line of sight (NLOS) communication. The coverage area ranges over a few tens of meters in this case. Larger cells are possible in outdoor environments. The microcells are deployed in urban areas to service high density usage. The BS is mounted at elevations lower than the skyscrapers to cover users at the street level. The cell coverage is confined within a few hundreds of meters. A microcellular environment is characterized predominantly by LOS propagation. Although, during mobility NLOS paths may result. To service a larger area spanning
a few kilometers, macrocells are used. Macrocells typically have the BS installed at
the building rooftop. Communication in macrocells follows both LOS and NLOS
paths. Megacells cover the largest area spanning hundreds of kilometers. Typically
satellites are used to achieve the large coverage. Due to high BS elevation, the
communication is predominantly LOS. The cell shape of megacells and macrocells
is represented by hexagons. However, due to lower antenna heights in femtocells,
picocells and microcells, the hexagonal approximation is inappropriate [2]. In these
cells, the cell geometry depends upon the specific deployment.

A wireless signal propagates between the BS and the MS through reflection,
diffraction and scattering from obstacles in the propagation path. The reception of
echoes of the transmitted signal is a common feature that is referred to as multipath
propagation. As a result, the received signal experiences path-loss, shadowing and
multipath fading. Path-loss represents the attenuation in the received signal power
at distances ranging from several meters to tens of kilometers from the base station.
The decay in received signal power due to path loss varies inversely as the square
of path length in free space. Shadowing is a slow variation in the local mean power
level. The interference from the obstacles in the transmission path that are much
larger than the wavelength of the radio wave, result in variability of the local mean
power level. Examples are buildings and hills in macrocells and smaller objects such
as vehicles in microcells. Empirical studies have shown that the local mean power
variation follows a log-normal distribution [2].

The distortions of the radio signal caused by channel obstructions are further
aggravated by the motion of the receiver. In this case, the frequency of the trans-
mitted signal is Doppler shifted at the receiver. The Doppler shift is a function of the speed of the MS. This effect can induce phase variations in the received signal that occur faster than the symbol rate, a phenomenon referred to as fast fading. At low mobile speeds, the phase variations cause the received signal variations to change over a slower time scale such that the channel can be assumed constant over a symbol duration.

On the short-time scale, where the mobile moves over a few meters range, the fluctuations of received signal amplitude about the local mean value occur due to multipath effects. The transmitted signal follows multiple trajectories before arriving at the receiver. The signals from these multiple paths may arrive within a symbol duration or across many successive symbols. In the former case, referred to as flat fading, the received signal varies within a symbol duration and is represented by the vector combination of the multipath components. Due to differences in path-length and direction of arrival of each multipath component, the received signal experiences destructive or constructive interference. The same situation occurs when the MS is stationary but the channel environment changes in time. If the multipath components are spread across more than one symbol, the fading is referred to as frequency-selective fading. This is a characteristic feature of wideband transmission, where the transmission bandwidth is larger or comparable to the channel bandwidth.

Due to time-variations, the received signal takes on random values in time. The mobile channel is therefore intrinsically time-varying, resulting in instantaneous changes in the signal to noise ratio (SNR) at the receiver. The effect of fading also results in inter-symbol interference (ISI) which causes dispersion in the time-profile
of the transmitted signal. During periods of destructive interference and high ISI, the SNR falls to a low value, causing symbol errors.

Due to uncertainty in isolating the factors that cause the received signal variations in time, a probabilistic and statistic approach is used for analyzing wireless channel characteristics. A discussion of these channel models is provided next.

1.2 Characterization of Wireless Channels

Channel features such as terrain, reflectors in the propagation paths and mobility in the radio propagation environment result in temporal variations in the attenuation of the received pulses. The received signal echoes also experience temporal delays and phase distortions. The models discussed in this section characterize the channel through the behavior of one or more of channel and system parameters.

1.2.1 Empirical Models

In 1964, Ossanna [3] proposed one of the first models that described the suburban outdoor environment. The signal properties in Ossanna’s model are evaluated using two different approaches, first using the Doppler shift in the received signal and the other using the standing wave pattern generated by vertically polarized transmitted waves and its reflections. The reflections are assumed to ensue from vertical stationary reflectors that are distributed uniformly in all directions. The effect of reflectors located far from the mobile is neglected. Ossanna’s model evaluates the power spectrum of the received signal as a function of mobile velocity and angle between incident ray and velocity vector. Due to mobility, the angle varies randomly


and is assumed to be uniformly distributed between $[0 : 2\pi]$. Once the spectrum is obtained for fixed values of the approach angle, the complete power spectrum is determined by averaging the spectrum over the range $[0 : 2\pi]$. Except at low frequencies, the spectra thus obtained is shown to conform closely with the spectrum determined from the experimental data collected in suburban residential environments.

Ossanna’s model was followed by Gilbert’s models [4] proposed in 1965. This work examined the effect of superposition of vertically polarized plane waves. Gilbert proposed three models, each based on different assumptions regarding the statistical properties of the received amplitude. A common feature in all of Gilbert’s models is uniformly distributed and mutually independent phases of the received waves. In the first model, it is shown that the electric field strength is Rayleigh distributed when inphase and quadrature components are assumed Gaussian distributed. The second and third models differ from the first model in distribution of received amplitude. The received amplitude in the second model is assumed constant and in the third model it is arbitrarily distributed. The uniformly distributed directions of arrival are chosen in both of the models mutually independently. The cumulative probability distribution function (PDF) of the energy density averaged over the combining waves is derived for all three models. It is shown that the distribution is the same for all the three cases in the limit of infinite number of combining waves. The models are applied to obtain auto and cross correlation coefficients between electric and magnetic fields and/or total energy density for fixed pair of observer locations.

Among the most widely used models today is the one proposed by Clarke [5] in 1968. Clarke’s model is applicable to propagation environments that are devoid
of any dominant transmission path and where the signal takes NLOS paths through
the process of scattering. The model assumes that multipath signals have equal like-
lihood of arrival from all directions, similar to Gilbert’s second model. In addition,
phase and angle of arrivals for each component wave are assumed to be statisti-
cally independent and uniformly distributed, unlike Gilbert’s models where angle of
arrivals were equally spaced on a unit circle. Properties of the received signal are
derived for vertically polarized plane wave transmission. It is assumed that the polari-
zation remains unchanged during transmission. Under these constraints, the inphase
and the quadrature components of the complex received signal follow a zero mean
Gaussian distribution and the received signal envelope is Rayleigh distributed. The
variance of inphase and quadrature components is the only parameter required for
complete characterization of the received envelope. Jakes [6] approximated Clarke’s
model by superposition of a large number of sinusoidal waves [6] considering fixed
Doppler frequency. Jakes simulator based on summation of sinusoids is widely used
for generation of Rayleigh distributed traces for simulation purposes.

When a LOS transmission path exists along with NLOS components, the prop-
erties of the received signal are influenced by the dominant LOS component. The
inphase and quadrature components now include a non-zero mean value. Under the
assumption of uniformly distributed phases, the received envelope of such a pro-
cess follows Rice probability distribution function [2]. The Ricean distribution was
introduced by Rice [7] to characterize the statistical properties of a sinusoidal sig-
nal corrupted by additive narrowband Gaussian noise. The Ricean fading signal is
characterized by the Rice factor $K$, which is the ratio between the power of the domi-
inant LOS signal and the variance of the NLOS multipath components. When the
dominant signal component weakens, the Ricean probability distribution function
approaches to the Rayleigh distribution.

For the channels characterized by fading environments that cannot be repre-
sented by Ricean or Rayleigh models, the Nakagami-m distribution has been pro-
posed [8]. The Nakagami-m distribution function was obtained based on the exper-
imental observations. This model makes no assumptions regarding the statistical
properties of received amplitude and phase. The Nakagami distribution is described
in terms of the average power of the received signal and a constant $m$. The value
of $m \geq 1/2$ is a function of average and instantaneous received signal power, and it
can be chosen depending upon the fading environment. For $m = 1$ the Nakagami
distribution is equal to the Rayleigh distribution. The Nakagami distribution ap-
proximates Ricean distribution for a suitable $m$ chosen as a function of Rice factor
$K$. All the aforementioned models deal with the small scale fading observed in the
vicinity of the receiver. For the large scale losses over T-R separation distances
path-loss models are considered.

In general, in free space environment the mean signal power for a given transmitter-
receiver separation $d$ is inversely proportional to $d^\alpha$ where $\alpha$ is the path loss exponent
that characterizes the space. This phenomenon is called path-loss. For free-space
$\alpha = 2$. For urban cellular environments, $\alpha$ varies between 2.7 and 3.5 and it is less
than 2 for indoor environments [9]. When the reflections from earth (approximated
as plane-surface) are considered, the power decays inversely as the fourth power of
d $[2]$. The decay rate is usually independent of carrier frequency for small angles of
reflection and $d$ much larger than the product of BS-MS elevation from the ground level. In the presence of man-made structures, the path-loss model becomes more complex and is environment specific. The path-loss cannot be specified by a power-law decay in such cases. For example, Okumura and Hata's models [2] characterize path-loss in macro cellular systems in the city of Tokyo as a function of BS and MS elevation and carrier frequency. The model is derived using the experimental data. Lee's model [2] characterizes path-loss in flat terrains. Using a correction factor, Lee's model can also be generalized for hilly terrain.

Different locations with the same T-R separation can exhibit a random variation about the average received local power. Therefore the local mean power can also be characterized probabilistically. Empirical studies have shown [2] that the average local power expressed in decibels is Gaussian distributed. Since the random variable average power is in logarithmic units, the probability distribution is a log-normal distribution.

The probability density function of the received signal considering the joint effects of path-losses and the local distortions is modeled by Suzuki [10]. Suzuki's model characterizes the received signal using the local path variations integrated over the path-loss. The path loss is assumed log-normally distributed and the local effects are approximated by choosing Rayleigh, Rice or Nakagami-m distribution based on the fading environment. Using the experimental results, Suzuki has shown that the Nakagami-m distribution characterizes the global path-strength variations better than Rayleigh or Rice distributions.

To jointly characterize path delays and amplitude variations, Saleh and Valen-
zuela [11] analyzed measurements taken in a slowly varying channel in an office environment. A vertically polarized omni-directional transmitter and receiver pair was used to measure the channel impulse response at a frequency of 1.5 GHz with a time resolution of 5 ns. The channel impulse response was averaged over a frequency bandwidth of 200 MHz. The measurements show that the spread of frequency averaged power profile in time increases for close T-R separation in hallways. The time spread widens when the measurements are made inside a room furnished using standard metallic office furniture and equipments. The frequency averaged power profile shows spikes spread over randomly spaced time intervals. Saleh and Valenzuela’s model characterizes these peaks by multipath echos arriving in clusters. The time between the clusters is modeled as a Poisson process with mean arrival rate $\Lambda$. The arrivals within the clusters form another independent Poisson process with mean arrival rate $\lambda >> \Lambda$. The probability distribution function of power gain normalized with mean power gain is assumed to be independent of associated delays. The power gain is exponentially distributed and therefore the voltage gain is Rayleigh distributed. An extensive survey of indoor channel models is presented in [12] and references therein.

1.2.2 Dynamic Channel Models

The empirical models described in the previous section characterize the steady-state features of the channel. Wireless channels are additionally characterized by temporal correlation. Therefore second-order statistics are important. The autocorrelation function of the Rayleigh fading channel is governed by the Bessel function
of zeroth order \( J_0(2\pi f_m \Delta t) \) as shown by Clarke [5]. The Bessel function arguments are a function of Doppler frequency shift \( f_m \) and \( \Delta t \) is the sampling time step. The short-range correlation properties of such a channel have been mapped to a stochastic process framework that will allow tractable analysis. The key requirement of such a model is in capturing features of the received signal variations at a time scale that will model the transitions to and from signal fade conditions. The earliest stochastic model is a 2-state Markov chain proposed by Gilbert [13] to characterize burst-errors in telephonic communication. Gilbert’s model classifies the channel into a good and a bad state. The bad state represents the times at which bit errors are encountered. Elliot [14] generalized Gilbert’s channel model by allowing each state to be characterized by a finite probability of error in good as well as bad channel state. The error probability is much lower in the good channel state. This generalization yields the well known Gilbert-Elliot channel model. Markov chains with multiple states \( N \) were proposed by Fritchman [15]. The \( N \)-state Markov chain is partitioned into two groups, one with \( k \) error states and another with \( N - k \) non-error states. The error statistics derived using Fritchman’s Markov models have been experimentally verified and found to represent fairly accurately the features of very high frequency channels [16] and frequency non-selective Ricean channels [17] in outdoor environments. Multiple state Markov chain models have been proposed for specialized cases of Rayleigh channels [18] [19] [20] [21] and Ricean channels [17] [22] [23].

In the state-space model, partitioning the signal into states is an important aspect of the model formulation. The partitioning of the signal into a state space representation can be accomplished by quantizing the power levels into thresholds
[22] or based on average received SNR [18] [21]. Babich et al. [23] obtained multiple states for the MC by sampling and quantizing the fading power with respect to a set of threshold values. In each case, the state of the Markov chain signifies a different average bit error rate (BER) of the channel and captures the temporal variations in the channel quality.

Markov channel models have been applied for characterization of various events in the channel. Zorzi [24] used Fritchman’s Markov model to characterize the event when the probability of signal-to-interference ratio falls below a threshold at a given time. Effects of channels with and without interleaving [20], characterization of block errors [25], rate control based on channel state [26] [27], signal detection using maximum likelihood sequence estimation and channel state estimation using maximum a-posteriori probability [28] are some of the applications that have been addressed using Markov chain models. Markov chains have also been used for evaluation of wireless protocols. Chocklingam et al. [29] evaluated a wireless access protocol that uses the busy/idle status of the BS to allow selective transmission of packets from the MS so that the packet losses due to collision and interference are minimized. The success or failure in decoding the received packet is cast in the form of a two state Markov chain model. The transition probabilities across the states are determined as the outcome of a comparison between the instantaneous SNR and a threshold SNR value in the presence of fading.

Autoregressive (AR) models have also been used in characterizing temporally varying channel features. Mychal et al. [30] have characterized the channel disturbance as an additive correlated noise process. An AR process with an a-priori chosen
order is used to model the channel disturbance with application toward signal detection. The AR parameters are obtained from covariance analysis of the received data sequence. Autoregressive modeling of the indoor wideband 0.9-1.1 GHz channel frequency response is discussed in [31]. The AR parameters are obtained from the measurements made in an office environment. A second order model with random complex valued poles is discussed in this work. The frequency response is generated by assuming either magnitude or angle of one of the poles constant and the rest as independent Gaussian processes that take on values only in a specified range. AR modeling of time varying channels with parameter estimation based on data is presented in [32] to characterize microwave channels in a LOS environment. An AR model for frequency selective fading channels is considered in [33]. The model is based on estimating the time-varying AR parameters from the correlation in the received sequence.

The aforementioned work on characterizing wireless channels is focused on the evaluation of channel performance statistics and in some cases verification of the approximation of channels through models using experimental data. Using Markov chains toward prediction of channel conditions in the future has attracted far less attention. Many researchers [19] [22] [25] have argued favorably to assert adequacy of first order Markov chains in modeling fading channels. The work of Tan et al. [34] however highlighted the inadequacy of Markov Chains for fast fading channels and for applications that require a longer memory for analysis. The second order AR model was found adequate in [31] whereas [32] used an AR model of 32nd order. The problems of wireless channel estimation and prediction are discussed next.
1.3 Channel Estimation

Channel estimation problems in communication channels involve the detection of channel features such as the channel impulse response and channel noise by making use of observations at the receiver. The process of using the received signal to estimate the channel impulse response, followed by recovery of the transmitted symbol is referred to as channel equalization [35]. The channel is typically modeled as a finite impulse response (FIR) linear predictive filter. The equalizer is an inverse filter for such a channel, designed to cancel out the ISI induced on the transmitted data by multipath arrivals. Hence, the equalizer coefficients can provide an estimate of the channel impulse response. When the transmitted data sequence is known, the equalizer coefficients can be obtained using the Weiner solution approach [35]. The Weiner solution minimizes the mean square of the estimation error between the transmitted and equalized data. The solution requires knowledge of the auto-correlation of the channel output sequence and cross-correlation between the input sequence and channel output. In general, the input symbols and hence the cross-correlation is unknown at the receiver. However, the filter coefficients can be obtained by periodic transmission of a known input symbol sequence. The known symbols are called pilot or training symbols and durations when they are transmitted are referred to as training periods. Estimates of equalizer coefficients are obtained during training periods using adaptive estimation techniques. The training periods are chosen to be long enough to ensure convergence of the estimation error. During normal data transmission, ISI effects are reduced using the equalizer coefficients computed during
training periods. Algorithms such as least mean square (LMS) and recursive least square (RLS) approaches are among the methods used for adaptive estimation of the equalizer [36]. Baissas and Sayeed [37] use pilot symbols to estimate the channel coefficients. A bank of matched filters are used to estimate each channel tap over a frame of transmitted symbols. The Weiner solution is used to evaluate the channel coefficients. In the case of fast time-variations in the channel, transmission of one pilot symbol instead of a training sequence periodically interspersed with data symbols has been proposed [33].

When the training symbols are not available, blind equalization techniques are used [38] [39] [40]. Most blind equalization techniques implement maximum likelihood strategies for channel and data estimation. These techniques formulate estimation problems in terms of the joint probability of received output sequence conditioned on knowing the input sequence and the channel impulse response (CIR). The maximum likelihood estimates of the CIR and the input data sequence are the values that maximize this joint probability density function.

Channel estimation techniques have been successfully used in slow fading channels with training periods and updates taking place at the time scale of channel variations. Generally, it is difficult to estimate the characteristic time scale on which training has to be invoked. To deal with this uncertainty, training sequences are transmitted periodically at the expense of increased overhead. This approach which is not tuned to aperiodic channel fading events, can be inefficient and not very productive when the training periods are not frequent enough.

The aforementioned channel estimation techniques are useful in only those
channels that vary at time scales much larger than the convergence time required by
the estimation algorithms. Estimation of impulse response of fast fading channels are
considered in [41]. Each tap of the channel impulse response is assumed sinusoidally
varying with time. The taps are modeled as linear combination of suitably chosen
basis functions and the temporal variation in the channel is expressed in terms of
the time-varying coefficients of expansion. The correlation of the channel output is
used to obtain the coefficients of the expansion. An analysis of estimation errors on
equalizer performance is treated for frequency-selective fast Rayleigh fading channels
in [42].

1.4 Channel Prediction

The temporal variation of signals on wireless channels is a highly correlated
process. This feature permits the prediction of channel behavior at a future time with
reference to conditions that exist at present. The linear-prediction of communication
channels using time-series models is presented in [43]. Using an FIR channel, the
model parameters are calculated using least-mean square analysis of the prediction
error in time and frequency domains. An AR model of order $p \geq 10$ for fading
channels that exhibit a maximum Doppler shift of 100 Hz is presented in [44] [45].
The AR coefficients are time varying. The model uses a sampling rate of twice the
Doppler frequency for the Rayleigh fading channel in order to achieve long range
prediction accuracy. The time varying auto-regressive coefficients are evaluated by
the minimum-mean square solution similar to the Weiner solution and updated every
observation interval using the least mean square (LMS) algorithm. In [46] a minimum
mean square solution using an exponentially decaying filter on past values is obtained to update the coefficients in time. In [47] the channel frequency response is modeled as a weighted sum of exponentials. The weights are the complex amplitudes and the exponential term is a function of angle of arrival of the incident wave at the mobile. The exponential term is estimated using singular value decomposition combined with eigen analysis of the observed channel behavior. The amplitude gains are obtained by model fitting techniques that minimize the squared error. Prediction performance with sampling interval of less than one wavelength is evaluated. The same approach is extended in [48] to determine wideband frequency response. The method presented in [48] formulates the CIR in terms of frequency dependent complex path loss factor and phase offset. Wideband channels in an environment with fixed scatterers are considered. An algorithm is developed to predict the impulse response by estimating the effective Doppler shift in each frequency band. Lindbom et al. [49] [50] have extended the classical Weiner filter solution to slowly time-varying channels. The estimated values of the CIR are obtained using second order AR moving average process. The covariance matrix of the regressors is assumed known a-priori.

1.5 Impact of Channel Prediction

With the knowledge of the channel model, the prediction of channel fading and designing training to take place at predicted intervals may be more appropriate. Knowledge of the channel conditions prior to transmission allows corrective measures to be taken during error states. The full channel capacity can be exploited during favorable channel conditions. The adaptive techniques based on the channel condi-
tions such as modulation [51] [52], channel coding, power-control [53], rate-control [26], error control [54] [55] are known to improve performance over time-varying channels. Adaptive coding applications are considered in [45]. The technique for estimation of decision-feedback equalizer coefficients based on the channel estimates instead of classical adaptation algorithms has been proposed [42]. Based on instantaneous estimates of channel impulse response (CIR), the channel can be equalized [41]. When the channel state information is available at the receiver, the cancellation of ISI can be improved for blind equalization techniques. Zhang and Kassam [21] have proposed selective combining of variable rate error codes transmitted through different channel states to attempt error correction before requesting retransmission. This approach is shown to increase throughput by reducing the decoding errors and requiring fewer retransmissions at the expense of increased delay at the receiver due to decoding overhead. The knowledge of channel state is utilized most effectively during slow fading by aborting retransmissions, and during fast fading by requesting re-transmission during error conditions. The channel state information is shown to improve the throughput performance by 11-18% under the slow-fading conditions for zero propagation and processing delays.

Information exchange between transmitter and the receiver is broadly organized into control information and data. In the applications involving highly compressed data, the control bits assume priority over the data as the correct reception of control bits is more critical towards efficient information decoding. The knowledge of channel information may be utilized in sequencing the information transmission based on its relative priority. The variation in the transmission redundancy based
on the channel conditions to increase the probability of correct detection and the priority based adaptive error protection are some techniques where prediction of the channel conditions may be useful.

The performance improvement through adaptive techniques implemented either at the transmitter or the receiver are governed by prediction accuracy of the channel fade levels.

1.6 Thesis Objectives

The work in this thesis deals with the NLOS communication present in femtocells, picocells, microcells and macrocells. Isotropic scattering is assumed and fading is characterized by the Rayleigh probability density function. Rayleigh fading channels exhibit heavy correlation and periodic trends in time. The correlation is influenced by variations in the Doppler frequency and data-rate. The research in this thesis aims at identifying predictive models for Rayleigh fading channels. These models are described with reference to channel parameters such as Doppler frequency, data-rate, transmitted signal power and additive noise power.

The predictive models aim at utilizing the channel correlation along with the past channel behavior to estimate channel conditions in the future. In order to obtain channel forecasts, the channel predictors should adapt to channel variations. The information regarding channel variations is obtained by sampling the channel periodically. Due to heavy correlation, successively sampled symbols experience almost the same distortion and hence provide almost identical information regarding the channel state. Sampling at larger intervals may provide more insight into the
channel behavior, but significant information may be lost if the sampling interval is too large. The work presented here aims at identifying the channel feature that can be utilized as a metric to evaluate the adequacy of sampling intervals. The metric can be used to identify an appropriate sampling frequency for the fading channels. The flat and frequency-selective and slow and fast fading cases on Rayleigh fading channels are considered. The model is applied for error control at the transmitter and the receiver.

The thesis is organized in six chapters. Chapter 2 presents statistical characterization of Rayleigh fading channels based on Clarke’s model. In this chapter, a method for partitioning the PDF of the received envelope in fade and non-fade states based on an error threshold is discussed. The threshold is evaluated as a function of signal to noise ratio and characterizes a fraction $0 < C < 1$ of total errors in the channel.

In chapter 3, the error threshold is applied towards the formulation of the Markov chain (MC) and AR models. The parameters of the models are derived as a function of channel characteristics. Performance analysis of the models demonstrates inadequacy of MC in representing the channel features on a short time scale. It is shown that at least a second order AR model is required to model the periodic correlation inherent in Rayleigh fading channels.

A sampling scheme is presented in chapter 4 to model the channel transitions originating in the fade state. The sampling lag is evaluated as a function of the channel parameters. The sampling interval is integrated with the AR model for channel predictions. It is shown that a slow as well as a fast fading channel can be
reasonably predicted using suitable sampling. It is demonstrated that a one step forecast using coarse samples is advantageous over multi-step forecast through fine samples. The model is applied for error control at the transmitter. Error control through interleaving is considered. Using the model, interleaving parameters are obtained as a function of channel features.

In chapter 5, the AR model is applied to forecast time varying multipath channels. A Kalman filter based prediction approach is presented. The performance of the model to cancel ISI on multipath channels is evaluated against the conventional decision feedback equalizer. It is shown that performance can be improved by > 25% when the CIR predicted by the model is used for equalization.

Chapter 6 summarizes the contributions and suggests future work.
CHAPTER 2

FADING CHANNELS

2.1 Introduction

Signals transmitted over wireless channels experience distortion in their amplitudes and phases due to additive and multiplicative effects of noise sources. The noise arises from sources internal to the end systems as well as from those present in the communication channel. Additive white Gaussian noise (AWGN) is typically used to characterize the cumulative effect of all contributing disturbances that cannot be explicitly determined. Multipath propagation effects due to delayed arrivals are manifest as both additive and multiplicative interference.

In a wireless channel, signal propagation between the source and the receiver can take place through the process of reflection, diffraction and scattering. The multipath components resulting from these propagation phenomena arrive at the receiver with varying attenuation and time delays. The attenuation of the mean signal level at the receiver with respect to its value at the transmitter can be characterized with the knowledge of the LOS transmitter and receiver (T-R) separation distance $d$. Typically the mean power level of the signal decays as $d^{-\alpha}$, where $\alpha$ is referred to as the path loss exponent. The magnitude of $\alpha$ is a function of the terrain features.
For free-space conditions, $\alpha = 2$, for urban areas $\alpha = 2.7$ to 3.5, and it is less than 2 for indoor environments [9]. When non-LOS conditions exist, this mean power level can be modeled as a log-normal random variable [2].

When the receiver is in motion or the channel environment is changing within distances of a few meters, the received power exhibits variations about the local mean power level. This type of variation is referred to as short-term fading, which is further characterized as flat or frequency-selective fading. The features distinguishing these types of fading are presented next.

### 2.2 Characterization of Fading Channels

Consider a low-pass signal $s(t)$ with envelope $a(t)$ and phase $\theta(t)$

$$s(t) = a(t)e^{j\theta(t)}$$  \hspace{1cm} (2.1)

When the signal is modulated using a carrier frequency $f_c$, a band-pass signal $s_b(t)$ results that may be represented as

$$s_b(t) = Re\left[ s(t)e^{j2\pi f_c t} \right]$$  \hspace{1cm} (2.2)

where, $Re[.]$ represents the real part of the argument. This signal takes multiple paths before arriving at the receiver. Let $n$ be the index of a signal path between the source and the receiver. The signal arrives at the receiver after a delay $\tau_n(t)$ observed at time $t$. The $n^{th}$ path also induces an amplitude scaling $a_n(t)$. The band-pass representation of the received multipath signal $r_b(t)$ is

$$r_b(t) = \sum_n a_n(t) s_b(t - \tau_n(t))$$  \hspace{1cm} (2.3)
Substituting Eq.(2.2) in Eq.(2.3) the received signal can be written as

\[
r_b(t) = Re \left[ \sum_n a_n(t) s(t - \tau_n(t)) e^{j2\pi f_c(t - \tau_n(t))} \right] \tag{2.4}
\]

Mobility induces further distortion in the received signal which is manifest as a Doppler shift in the carrier frequency. The Doppler frequency shift \(D_n(t)\) for the \(n^{th}\) arriving path is a function of mobile velocity \(v(t)\) and angle \(\beta_n(t)\) between the incident ray and the velocity vector. \(D_n(t)\) is given as

\[
D_n(t) = \frac{v(t)}{\lambda_c} \cos \beta_n(t) \tag{2.5}
\]

\[
= f_m(t) \cos \beta_n(t)
\]

where \(\lambda_c\) is the wavelength of the propagating wave and \(f_m(t) = v(t)/\lambda_c\) is the Doppler frequency. When \(v(t)\) is constant in time, the Doppler frequency is a fixed value. Due to the Doppler effect, the frequency of the received signal is altered as \(f_c \pm |D_n(t)|\) depending whether the motion of the mobile is towards or away from the observer. The effect of Doppler shift is included by replacing \(f_c\) by \(f_c + D_n(t)\) in Eq.(2.4). Thus the band-pass received signal under mobility conditions is

\[
r_b(t) = Re \left[ \sum_n a_n(t) s(t - \tau_n(t)) e^{j2\pi (f_c + D_n(t))(t - \tau_n(t))} \right] \tag{2.6}
\]

\[
= Re \left[ \left( \sum_n a_n(t) s(t - \tau_n(t)) e^{-j2\pi \phi_n(t)} \right) e^{j2\pi f_c t} \right]
\]

where,

\[
\phi_n(t) = |f_c + D_n(t)| \tau_n(t) - t \cdot D_n(t) \tag{2.7}
\]

The complex low-pass representation of the received signal is obtained after demodulation. Factoring out the term \(e^{j2\pi f_c t}\) from Eq.(2.6) yields

\[
r(t) = \sum_n a_n(t) s(t - \tau_n(t)) e^{-j2\pi \phi_n(t)} \tag{2.8}
\]
Under the assumption that the amplitudes and phase variations in time are wide-
sense stationary (WSS), the first and the second moments of $r(t)$ are constant in
time. Therefore, in Eq.(2.8) the following substitutions can be made,

$$D_n(t) = D_n, \quad \tau_n(t) = \tau_n \quad a_n(t) = a_n$$

(2.9)

The complex low-pass received signal under WSS conditions is

$$r(t) = \sum_n a_n s(t - \tau_n) e^{-j2\pi \phi_n(t)}$$

(2.10)

where,

$$\phi_n(t) = [f_c + D_n] \tau_n - t D_n$$

(2.11)

Comparing the transmitted signal in Eq.(2.1) and received signal in Eq.(2.10)
the complex low-pass channel impulse response (CIR) $h(\tau; t)$ can be inferred to be

$$h(\tau; t) = \sum_n a_n \delta(\tau - \tau_n) \ e^{-j2\pi \phi_n(t)}$$

(2.12)

where $\phi_n(t) = [f_c + D_n] \tau_n - tD_n$

The function $h(\tau; t)$ represents the response of the channel at time $t$ to an impulse
applied at $t - \tau$. The channel features can be completely characterized by examining
the correlation properties in time and frequency.

The channel correlation with respect to multipath arrivals $m$ and $n$ at time $t$
and $t + \Delta t$ is given for complex CIR [56] by

$$\rho_h(\tau_n, \tau_m; \Delta t) = E \left[ h^*(\tau_n; t) \ h(\tau_m; t + \Delta t) \right]$$

(2.13)

where $h^*$ represents the complex conjugate. Since the distinct arrival paths have
independent amplitudes and delays, the correlation simplifies to

$$\rho_h(\tau_n, \tau_m; \Delta t) = \rho_h(\tau_n; \Delta t) \ \delta(\tau_n - \tau_m)$$

(2.14)
The features of the autocorrelation $\rho_h(\tau_n; \Delta t)$ as a function of $\Delta t$ are investigated next. When $\Delta t = 0$, the correlation function provides the average power as a function of path delay $\tau_n$ as

$$\rho_h(\tau_n; 0) = E\left[ h^*(\tau_n; t) \ h(\tau_n; t) \right] \quad (2.15)$$

This equation is often referred to as the multipath or power delay profile (PDP). The shape and width of the PDP are of interest in understanding the inter-symbol Interference (ISI) caused by multipath arrivals. It cannot always be assumed that the PDP exhibits monotonic decay. Representing the delay $\tau_n$ generally as $\tau$, the mean $\bar{\tau}$ and variance $\sigma^2_{\tau}$ are characteristic parameters, determined from the PDP as

$$\bar{\tau} = \frac{\int_\tau \tau \rho_h(\tau; 0) \ d\tau}{\int_\tau \rho_h(\tau; 0) \ d\tau} \quad (2.16)$$

$$\sigma^2_{\tau} = \frac{\int_\tau \tau^2 \rho_h(\tau; 0) \ d\tau}{\int_\tau \rho_h(\tau; 0) \ d\tau} - (\bar{\tau})^2$$

The standard deviation $\sigma_{\tau}$ is referred to as the channel delay spread and is a measured characteristic of the channel.

The characterization of channel in frequency domain is undertaken by considering the transfer function

$$H(f; t) = \int_0^\infty h(\tau; t) \ e^{-j2\pi f\tau} \ d\tau \quad (2.17)$$

$H(f; t)$ inherits the random characteristics of $h(\tau; t)$. The correlation in the frequency domain is

$$\rho_H(f_1, f_2; \Delta t) = E\left[ H^*(f_1; t) \ H(f_2; t + \Delta t) \right] \quad (2.18)$$
and can be obtained by taking the Fourier transform of Eq. (2.14) as follows

\[
\rho_H (f_1, f_2; \Delta t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho_H (\tau_n; \Delta t) \delta (\tau_n - \tau_m) e^{j2\pi (f_1\tau_n - f_2\tau_m)} d\tau_n d\tau_m
\]

\[
= \int_{-\infty}^{\infty} \rho_H (\tau_n; \Delta t) e^{j2\pi (f_1 - f_2)\tau_n} d\tau_n
\]

\[
= \rho_H (\Delta f; \Delta t)
\]

where \( \Delta f = f_1 - f_2 \)

Thus, the frequency correlation may be expressed in terms of the frequency spacing \( \Delta f \) as

\[
\rho_H (f_1, f_2; \Delta t) = \rho_H (\Delta f; \Delta t) \tag{2.19}
\]

The bandwidth of this frequency domain function provides a measure of the coherence \( \Delta f_c \) produced by the channel in the frequency variable. The reciprocity in time and frequency domains leads to the relation

\[
\Delta f_c \propto \frac{1}{\sigma_r} \tag{2.20}
\]

If the transmitted signal bandwidth \( \Delta f_s \) exceeds the coherence bandwidth of the channel \( \Delta f_c \), frequencies of the transmitted signal are selectively distorted. When \( \Delta f_s < \Delta f_c \), one encounters the frequency insensitive or a flat fading channel.

A further characterization of the channel may be made as slow or fast fading by considering non-zero values of the time shift parameter \( \Delta t \) in the frequency correlation function \( \rho_H (\Delta f; \Delta t) \). As discussed before, mobility induces Doppler spread in the transmitted frequency spectrum. The Doppler spread is directly proportional to the velocity of the mobile. Consider the Fourier transform of the frequency correlation function \( \rho_H (\Delta f; \Delta t) \) with respect to \( \Delta t \). The result is a spectral density in
the Doppler frequency variable $\lambda$

$$S_H(\Delta f; \lambda) = \int_{-\infty}^{\infty} \rho_H(\Delta f; \Delta t)e^{-j2\pi\lambda\Delta t} d(\Delta t) \quad (2.21)$$

Analyzing the spectrum at a fixed frequency, that is, $\Delta f = 0$, $S_H(0; \lambda)$ provides the signal intensity variations as a function of $\lambda$. This function is referred to as the Doppler power spectrum of the channel. The spectrum assumes the form of a delta function when $\rho_H(0; \Delta t) = 1$ for all $\Delta t$ indicating that no spectral broadening occurs for time invariant channels. The characteristic bandwidth of $S_H(0; \lambda)$ is the Doppler spread $B_d$. This parameter is inversely proportional to the channel coherence time $\Delta t_c$, where $\Delta t_c$ is the standard deviation of $\rho_H(0; \Delta t)$. Channels changing slowly in time are characterized by large coherence times, resulting in small Doppler spreads. Fast fading channels have smaller coherence times and correspondingly large Doppler spread values.

Next, the signal received under Rayleigh fading conditions is characterized in terms of its amplitude and phase distributions and temporal features such as level crossing rates and autocorrelation function.

2.3 Clarke’s Rayleigh Fading Model

The multipath channel proposed by Clarke [5] assumes that the transmitted signal arrives at the receiver from all directions with equal probability. The relative delays associated with each multipath component are small enough to be considered as lying within the duration of a single transmitted symbol. The amplitudes and phases of different multipath components are independent of each other. The received
The band-pass signal given in Eq. (2.6) is

\[ r_b(t) = \text{Re} \left[ \sum_n a_n(t) s(t - \tau_n) e^{-j2\pi \phi_n(t)} e^{j2\pi f_c t} \right] \quad (2.22) \]

Expanding the above equation using Euler’s identity and the WSS assumption in Eq. (2.9) gives

\[ r_b(t) = \sum_n a_n(t) s(t - \tau_n) \cos(2\pi \phi_n(t)) \cos(2\pi f_c t) \quad (2.23) \]

\[ - \sum_n a_n(t) s(t - \tau_n) \sin(2\pi \phi_n(t)) \sin(2\pi f_c t) \]

Using the substitutions

\[ y(t) = \sum_n a_n(t) s(t - \tau_n) \cos(2\pi \phi_n(t)) \quad (2.24) \]

\[ z(t) = \sum_n a_n(t) s(t - \tau_n) \sin(2\pi \phi_n(t)) \]

the signal \( r_b(t) \) can be written as

\[ r_b(t) = y(t) \cos(2\pi f_c t) - z(t) \sin(2\pi f_c t) \quad (2.25) \]

From Eq. (2.10), it can be inferred that the variable \( y(t) \) is the inphase and \( z(t) \) is the quadrature component of the received low-pass signal \( r(t) \). The autocorrelation of the signal \( r_b(t) \) is given by

\[ \rho_{r_b r_b}(\Delta t) = E \left[ r_b(t) r_b(t + \Delta t) \right] \quad (2.26) \]

Substituting Eq. (2.25) in Eq. (2.26) and using \( 2\pi f_c = \omega_c \) yields

\[ \rho_{r_b r_b}(\Delta t) = E \left[ \frac{y(t) y(t + \Delta t)}{2} [\cos(\omega_c t + \omega_c \Delta t) + \cos(\omega_c t)] \right] \]

\[ - E \left[ \frac{y(t) z(t + \Delta t)}{2} [\sin(\omega_c t + \omega_c \Delta t) + \sin(\omega_c t)] \right] \]

\[ - E \left[ \frac{z(t) y(t + \Delta t)}{2} [\sin(\omega_c t + \omega_c \Delta t) - \sin(\omega_c t)] \right] \]

\[ - E \left[ \frac{z(t) z(t + \Delta t)}{2} [\cos(\omega_c t + \omega_c \Delta t) - \cos(\omega_c t)] \right] \]
Rearranging the terms gives

\[
\rho_{r_y r_y}(\Delta t) = \left[ \frac{\rho_{yy}}{2} - \frac{\rho_{zz}}{2} \right] \cos(\omega_c t + \omega_c \Delta t) + \left[ \frac{\rho_{yy}}{2} + \frac{\rho_{zz}}{2} \right] \cos(\omega_c \Delta t) \tag{2.27}
\]

\[
- \left[ \frac{\rho_{yz}}{2} + \frac{\rho_{zy}}{2} \right] \sin(\omega_c t + \omega_c \Delta t) - \left[ \frac{\rho_{yz}}{2} - \frac{\rho_{zy}}{2} \right] \sin(\omega_c \Delta t)
\]

where \(\rho_{yy}(\Delta t)\), \(\rho_{zz}(\Delta t)\) are the autocorrelation and \(\rho_{yz}(\Delta t)\) and \(\rho_{zy}(\Delta t)\) are the cross-correlation functions of inphase and quadrature components. Under the assumption of WSS conditions on \(r_h(t)\), the expectation in the above equation should be only a function of the time interval \(\Delta t\). The conditions necessary for the relations given in Eq.(2.27) to be wide-sense stationary are examined next by evaluating the auto and cross correlation functions of the inphase and quadrature signals.

Using the substitution

\[
\alpha_n = a_n s(t - \tau_n)
\]

in Eq.(2.24), the autocorrelation function of the inphase component can be written as

\[
\rho_{yy}(\tau) = E \left[ \sum_n \sum_m \alpha_n \alpha_m \cos(2\pi \phi_n(t)) \cos(2\pi \phi_m(t + \tau)) \right] \tag{2.29}
\]

\[
= \sum_n \sum_m E[\alpha_n \alpha_m \cos(2\pi \phi_n(t)) \cos(2\pi \phi_m(t + \tau))]
\]

The summation can be expanded and expectation of each term can be evaluated individually. Assuming phase and amplitudes are mutually independent, the expectation \(E[\alpha_n \alpha_m]\) can be separated from the rest of the terms in Eq.(2.29).

Consider the following expectation for the case when \(m \neq n\)

\[
E[\cos(2\pi \phi_n(t)) \cos(2\pi \phi_m(t + \Delta t))] = E[\cos(2\pi \phi_n(t))] \quad E[\cos(2\pi \phi_m(t + \Delta t))]
\]

\[
(2.30)
\]
if the phases of distinct multipath components are independent. When \( \phi_n(t) \) is uniformly distributed, the PDF of \( \cos(2\pi \phi_n(t)) \) can be obtained by using the transformation of random variables \([57]\). The function \( u = \cos \phi \) is distributed as

\[
f_U(u) = \frac{-1}{\pi \sqrt{1 - u^2}} \quad 0 < u \leq 1 \tag{2.31}
\]

Applying this distribution, the expectation \( E[\cos(2\pi \phi_n(t))] = 0 \). Therefore the autocorrelation function (ACF) simplifies to,

\[
\rho_{yy}(\Delta t) = \sum_n E \left[ \alpha_n^2 \cos(2\pi \phi_n(t)) \cos(2\pi \phi_n(t + \Delta t)) \right] \tag{2.32}
\]

Converting the product of two cosines in the above equation to the sum of two cosines with arguments \( \phi_n(t) + \phi_n(t + \Delta t) \) and \( \phi_n(t + \Delta t) - \phi_n(t) \), the expected value \( E[\cos(\phi_n(t) + \phi_n(t + \Delta t))] = 0 \) using the PDF in Eq.(2.31) again. Therefore the ACF is now

\[
\rho_{yy}(\Delta t) = \frac{1}{2} \sum_n E \left[ \alpha_n^2 \cos 2\pi (\phi_n(t + \Delta t) - \phi_n(t)) \right] \tag{2.33}
\]

Using value of \( \phi_n(t) \) from Eq.(2.11) the cosine argument \( \phi_n(t + \Delta t) - \phi_n(t) \) can be evaluated to be a constant

\[
\phi_n(t + \Delta t) - \phi_n(t) = D_n \Delta t \tag{2.34}
\]

It follows that

\[
\rho_{yy}(\Delta t) = \frac{1}{2} \sum_n E(\alpha_n^2) \ E[\cos(2\pi D_n \Delta t)] \tag{2.35}
\]

Assuming that the Doppler shift experienced by each received component \( D_n = D \) is a constant, the summation reduces to

\[
\rho_{yy}(\Delta t) = \frac{E[\cos(2\pi D \Delta t)]}{2} \sum_n E(\alpha_n^2) \ O_p \ \frac{\eta_p}{2} \tag{2.36}
\]
where \( \frac{\Omega_p}{2} = \sum \frac{E(\alpha_i^2)}{2} \) is the average power in the composite multipath signal. Substituting the value of the Doppler frequency from Eq. (2.5)

\[
E \left[ \cos(2\pi D \Delta t) \right] = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos(2\pi f_m \Delta t \cos \beta) d\beta \quad (2.37)
\]

The above integral can also be written as [58]

\[
\rho_{yy}(\Delta t) = \frac{\Omega_p}{2} J_0(2\pi f_m \Delta t) \quad (2.38)
\]

where \( J_0(.) \) is the zeroth order Bessel function of first kind. Following steps similar to Eq.(2.29)-(2.36) it can be shown that

\[
\rho_{zz}(\Delta t) = \rho_{yy}(\Delta t) \quad (2.39)
\]

\[
\rho_{yz}(\Delta t) = -\rho_{zy}(\Delta t) = 0
\]

The relationships in Eq.(2.39) satisfy the conditions in Eq.(2.27) deemed necessary for the received signal to be WSS. Substituting \( \Delta t = 0 \) in Eq.(2.38) and using relationship in Eq.(2.39), the variance of the Gaussian distributed inphase and quadrature components can be obtained as

\[
\sigma_Y^2 = \sigma_Z^2 = \frac{\Omega_p}{2} \quad (2.40)
\]

The power spectrum is the Fourier transform of the autocorrelation function. The spectrum of \( y(t) \) and \( z(t) \) is given by [2]

\[
S_Z(f) = S_Y(f) = \frac{\Omega_p}{4\pi f_m} \left[ 1 - \left( \frac{f}{f_m} \right)^2 \right]^{-1/2} \quad |f| \leq f_m \quad (2.41)
\]

Fig. 2.1(a) shows the normalized ACF against \( n \) where \( \Delta t = n \Delta t_s, n = 0, 1, 2, \cdots \). The symbol interval \( \Delta t_s = 72 \times 10^{-6} \) which corresponds to a data-rate of
14 kbps. The autocorrelation is evaluated for $f_m = 90$ Hz and 180 Hz using Eq. (2.38). The correlation function exhibits damped periodic behavior. The main lobe decay rate increases with Doppler frequency. Fig. 2.1(b) shows the power spectral density for inphase component of the low-pass signal. The minimum power occurs at the center and peaks at $\pm f_m$. The band-pass power-spectrum is identical except for a shift equal to the carrier frequency $f_c$.

### 2.3.1 Probability Density Function of Received Envelope

Substituting Eq. (2.24) in Eq. (2.10) the complex low-pass received signal $r(t)$ can also be represented using the inphase and quadrature functions $y(t)$ and $z(t)$ respectively as,

$$ r(t) = y(t) + jz(t) = x(t)e^{j\varphi(t)} \quad (2.42) $$

where

$$ x(t) = \sqrt{y^2(t) + z^2(t)} \quad \text{and} \quad \varphi(t) = \tan^{-1} \frac{z(t)}{y(t)} \quad (2.43) $$
Applying the central limit theorem in the limit as \( n \to \infty \) in Eq.(2.24), the inphase and quadrature components can be assumed to be Gaussian distributed. In non-LOS conditions, the absence of a dominant signal renders the distribution to have a zero mean. In Eq.(2.40) the inphase and quadrature components are shown to exhibit equal variance of \( \frac{\Omega_n}{2} \) under isotropic scattering conditions. Therefore, the PDF of \( y(t) \) and \( z(t) \) can be written as

\[
    f_Y(y) = \frac{1}{\sqrt{\pi \Omega_p}} e^{-\frac{y^2}{\Omega_p}} \tag{2.44}
\]

\[
    f_Z(z) = \frac{1}{\sqrt{\pi \Omega_p}} e^{-\frac{z^2}{\Omega_p}}
\]

It can be observed from Eq.(2.39) that inphase and quadrature components are uncorrelated. Gaussian distributed uncorrelated random variables are also independent. The positive square root of sum of squares of two independent zero mean Gaussian variates can be shown to be Rayleigh distributed [57]. Therefore the envelope \( x(t) \) is Rayleigh distributed as

\[
    f_X(x) = \begin{cases} 
        \frac{2x}{\Omega_p} e^{-\frac{x^2}{\Omega_p}} & x \geq 0 \\
        0 & \text{otherwise}
    \end{cases}
\]

The cumulative density of \( x(t) \) is

\[
    F_X(x) = 1 - e^{-\frac{x^2}{\Omega_p}} \tag{2.45}
\]

The mean of the Rayleigh distribution is given by

\[
    x_{\text{mean}} = E[x] = \int_0^\infty x f_X(x) dx \tag{2.46}
\]

\[
    = \sqrt{\frac{\Omega_p}{2}} \frac{\pi}{2}
\]
The variance of Rayleigh distribution \( \sigma_X^2 \) is

\[
\sigma_X^2 = E[x^2] - (E[x])^2 = \frac{\Omega_p}{2} \left(2 - \frac{\pi}{2}\right)
\]

(2.47)

where \( E[x^2] = \int_0^\infty x^2 f_X(x) dx \) is the second moment of \( x(t) \). Fig. 2.2 shows the Rayleigh PDF for inphase and quadrature components with \( \Omega_p = 1 \). The values close to zero occur under destructive interference and the large values of envelope are the result of constructive interference.

2.4 Fading Statistics

2.4.1 Level Crossing Rate and Average Fade duration

The level crossing rate is the frequency at which the received signal crosses a specified level \( R \). The level crossing rate is a function of Doppler frequency and signal strength. The number of level crossings per second \( N_R \) for the Rayleigh fading
signal is given by [2]

\[ N_R = \sqrt{2\pi f_m \lambda e^{-\lambda}} \]  

(2.48)

where \( \lambda = \frac{R}{R_{rms}} \) and \( R_{rms} \) is the root-mean square power.

The average fade duration is the average period that the signal remains below a specified signal level \( R \). The amplitudes below \( R \) are associated with fading. The level crossing rate determines the number of times the fade state is entered and the average duration spent below the level \( R \). The average fade duration is [2]

\[ T = \frac{e^{-\lambda^2} - 1}{\lambda f_m \sqrt{2\pi}} \]  

(2.49)

2.5 Characterization of Additive Noise

The interference effects from the environment, including effects of thermal noise in electronic devices and other sources of electromagnetic radiation are modeled as independent noise sources. Their cumulative effect is represented by a Gaussian process. A Gaussian random variate \( \eta(t) \) added to the signal at the input to the receiver models the signal distortion due to these effects. It is assumed that the additive noise is a white Gaussian process (AWGN). In such a case, all frequencies have the same power. The power spectrum \( S_N(f) \) is given by,

\[ S_N(f) = \frac{N_0}{2}, \quad -\infty < f < \infty \]  

(2.50)

The notation \( \frac{N_0}{2} \) arises from quantum mechanics, where \( N_0 = kT \), \( k \) and \( T \) representing Boltzmann's constant and temperature in degrees Kelvin. The value \( \frac{N_0}{2} \)
represents the maximum value of the spectrum of thermal noise which occurs at zero frequency. The correlation of the AWGN process is obtained by taking the inverse Fourier transform of the power spectrum,

\[ \rho_{\eta\eta}(\tau) = \int_{-\infty}^{\infty} S_N(f) e^{j2\pi f \tau} \, df \]

\[ = \frac{N_0}{2} \delta(\tau) \]  

(2.51)

The noise variance obtained by substituting \( \tau = 0 \) in Eq.(2.51) is infinite. To avoid the mathematical intractability of infinite variance processes, the noise is assumed to have a finite bandwidth that is typically much larger than the spectrum of the transmitted signal. The noise spectrum in the pass band \([f_1, f_2]\) is represented as

\[ S_N(f) = \begin{cases} \frac{N_0}{2}, & |f_1| \leq |f| \leq |f_2| \\ 0 & \text{otherwise} \end{cases} \]  

(2.52)

The corresponding ACF is,

\[ \rho(\tau) = \int_{-f_2}^{-f_1} \frac{N_0}{2} e^{j2\pi f \tau} \, df + \int_{f_1}^{f_2} \frac{N_0}{2} e^{j2\pi f \tau} \, df \]

\[ = \frac{N_0}{2} \left[ \frac{\sin(2\pi f_2 \tau) - \sin(2\pi f_1 \tau)}{\pi \tau} \right] \]  

(2.53)

The variance of the band-pass noise can be evaluated by substituting \( \tau = 0 \) in Eq.(2.53)

\[ \rho_{\eta\eta}(0) = \frac{N_0}{2} \frac{\sin(\pi \tau(f_2 - f_1))}{\pi \tau} \]

\[ = \frac{N_0}{2} 2(f_2 - f_1) \]  

(2.54)

The factor \( 2(f_2 - f_1) \) is often suppressed in the band-pass representation and the variance is simply represented as \( \frac{N_0}{2} \). The amplitude variations of the noise are
distributed as a zero-mean Gaussian process with variance $\frac{N_0}{2}$. The noise PDF is given as,

$$f_N(\eta) = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{\eta^2}{N_0}} \quad (2.55)$$

The quality of a communication channel that is subject to AWGN is defined in terms of the signal-to-noise ratio (SNR). SNR is the ratio of average signal power to the average noise power. It is typically given in decibels. The SNR may be distinguished as transmitted, received or instantaneous SNR, based on whether the average power of the transmitted signal, average power of the received signal or instantaneous power in the received signal is considered. Representing the average signal power per bit as $\epsilon_b$, the SNR is,

$$SNR = 10 \log_{10} \frac{2\epsilon_b}{N_0} \quad dB \quad (2.56)$$

### 2.6 Probability Density Function of the Received Signal

In presence of noise, the received low-pass signal $r(t)$ in Eq.(2.42) can be expressed as the sum of $x(t)e^{j\varphi(t)}$ and $\eta(t)$ as

$$r(t) = x(t)e^{j\varphi(t)} + \eta(t) \quad (2.57)$$

where $x(t)$ is the Rayleigh distributed signal envelope and $\eta(t)$ is Gaussian distributed noise. Under coherent detection conditions $\varphi(t)$ is known. Since the noise and the fading are statistically independent processes, the PDF $f_R(r \mid \varphi)$ of the received signal can be represented as

$$f_R(r \mid \varphi) = f_X(x) * f_N(\eta) \quad (2.58)$$
where $\ast$ represents the convolution operation. The PDF $f_R(r \mid \varphi)$ can be obtained either by numerical integration or by evaluating the inverse Fourier transform

$$f_R(r \mid \varphi) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F_R(j\xi) \, e^{j\xi r} \, d\xi \tag{2.59}$$

where

$$F_R(j\xi) = F_X(j\xi) \, F_N(j\xi) \tag{2.60}$$

and $F_N(j\xi)$ and $F_X(j\xi)$ are Fourier transforms of Gaussian and Rayleigh PDF respectively given by

$$F_N(j\xi) = \int_{-\infty}^{\infty} f_N(\eta) \, e^{-j\xi \eta} \, d\eta \tag{2.61}$$

$$F_X(j\xi) = \int_{-\infty}^{\infty} f_X(x) \, e^{-j\xi x} \, dx$$

It is shown in Appendix-A that using Eq.(2.59) the PDF of the received signal can be obtained as

$$f_R(r \mid \varphi) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp \left[ -\frac{\xi^2}{8} \frac{2N_0 + \Omega_p}{j\xi r} + j\xi r \right] \mathcal{D}_{-2} \left( j \xi \sqrt{\frac{\Omega_p}{2}} \right) \, d\xi \tag{2.62}$$

where

$$\mathcal{D}_{-2}(\psi) = e^{\frac{\psi^2}{2}} \sqrt{\frac{\pi}{2}} \left[ e^{\frac{-\psi^2}{2}} \sqrt{\frac{\pi}{2}} - \psi \left( 1 - \Phi \left( \frac{\psi}{\sqrt{2}} \right) \right) \right] \tag{2.63}$$

$\Phi(\psi)$ is the error function defined as [59](p.938),

$$\Phi(\psi) = \frac{2}{\sqrt{\pi}} \int_{0}^{\psi} e^{-t^2} \, dt \tag{2.64}$$

The closed form solution to the integral in Eq.(2.62) can be evaluated for the special case when $\frac{N_0}{2} = 1$. As shown in the Appendix-A, for $\frac{N_0}{2}$ the PDF of the received
signal is given by

\[ f_R(r \mid \varphi) = \frac{1}{2 + \Omega_p} e^{-\frac{(4\Omega_p)^2}{4(\Omega_p^2 + \eta^2) + \frac{\psi^2}{4}}} B(\psi) \]  \hspace{2cm} (2.65)

where \[ B(\psi) = \sqrt{\frac{\pi}{2}} e^{-\frac{\psi^2}{2}} - \psi \left( 1 - \Phi \left( \frac{\psi}{\sqrt{2}} \right) \right), \quad \psi = -r \sqrt{\frac{\Omega_p}{2 + \Omega_p}} \]

It follows from Eq.(2.62) that the PDF \( f_R(r \mid \varphi) \) depends upon the noise variance \( \frac{N_0}{2} \) and \( \Omega_p \) and not the transmitted SNR as one might expect.

![Figure 2.3: PDF of received signal \( f_R(r \mid \varphi) \), Rayleigh envelope \( f_X(x) \), AWGN \( f_N(\eta) \) for \( \Omega_p = \frac{N_0}{2} = 1 \)]

Fig. 2.3 shows probability distribution functions \( f_X(x) \), \( f_N(\eta) \) and \( f_R(r \mid \varphi) \) for \( \Omega_p = 1 \) and \( \frac{N_0}{2} = 1 \). The PDF \( f_X(x) \) is obtained from Eq.(2.45), \( f_N(\eta) \) from Eq.(2.55) and \( f_R(r) \) from Eq.(2.65).

### 2.7 Probability of Detection Error

Detection errors at the receiver for a given channel are a function of the modulation scheme used during transmission. In the modulated signal, either amplitude
or frequency or phase of the carrier wave or a combination of these are varied to represent the transmitted symbols. Binary phase shift keying (BPSK) modulation is applied in this work. In BPSK modulation, the phase of the transmitted signal is altered so that positive and negative amplitudes of the transmitted pulses represent binary 1 and 0 respectively. The detection errors occur when the signal is modified by the channel noise such that the received symbols bear a sign that is opposite to what is transmitted. In this section, the probability of error is derived for an AWGN channel. The analysis is extended to wireless fading channels.

2.7.1 Error Probability in AWGN Channels

Consider a signal transmitted using BPSK over a channel with SNR=$2\epsilon_b/N_0$. The transmitted signal is $s(t) = \pm \sqrt{\epsilon_b}$ and the noise variance is $N_0/2$. Defining symbols $s_1$ and $s_2$ as

$$s(t) = \begin{cases} 
\sqrt{\epsilon_b} = s_1 \\
-\sqrt{\epsilon_b} = s_2 
\end{cases}$$

(2.66)

Let $f_R(r \mid s_1)$ and $f_R(r \mid s_2)$ be the PDF of the received signal conditioned on $s_1$ and $s_2$ being transmitted respectively. Using the maximum-likelihood criteria, the decision boundary for a BPSK signal is $r = 0$ [35]. Therefore, the conditional error probabilities can be obtained as

$$P(e \mid s_1) = P(-\infty < r \leq 0 \mid s_1) = \int_{-\infty}^{0} f_R(r \mid s_1) \, dr$$

(2.67)

$$P(e \mid s_2) = P(0 < r \leq \infty \mid s_2) = \int_{0}^{\infty} f_R(r \mid s_2) \, dr$$
Due to symmetry, $P(e|s_1) = P(e|s_2)$. Since $s_1$ and $s_2$ are equally likely, the average probability of error $P_e$ is

\[
P_e = \frac{1}{2} \left( \int_{-\infty}^{0} f_R(r \mid s_1) \, dr + \int_{0}^{\infty} f_R(r \mid s_2) \, dr \right)
\]

\[
= \int_{0}^{\infty} f_R(r \mid s_1) \, dr
\]  

(2.68)

For the AWGN channel, the received low-pass signal is given by $r(t) = s(t) + \eta(t)$. Using the noise PDF in Eq.(2.55), the PDF of the received signal conditioned on the transmitted signal can be obtained as

\[
f_R(r \mid s_1) = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(r-s)^2}{2N_0}}
\]  

(2.69)

Substituting Eq.(2.69) in (2.68) and solving, the probability of error can be evaluated as

\[
P_e = \int_{0}^{\infty} e^{-\frac{(r-s)^2}{2N_0}} \, dr
\]

\[
= Q\left(\sqrt{\frac{2\epsilon_b}{N_0}}\right)
\]

\[
= Q\left(\sqrt{2\gamma_b}\right)
\]

where $\gamma_b = \frac{\epsilon_b}{N_0}$ and $Q(z) = \int_{z}^{\infty} e^{-t^2} \, dt$ is the complementary error function.

### 2.7.2 Error Probability in Fading Channels

Consider the fading signal given by Eq.(2.57). Since $x(t)$ varies with time the signal transmitted with SNR=$2\epsilon_b/N_0$ is received with a time-varying instantaneous SNR $\gamma_b$. Under coherent detection the expected value of the received SNR $\bar{\gamma}_b$ is

\[
\bar{\gamma}_b = \frac{2\epsilon_b}{N_0} E[x^2]
\]  

(2.71)
where \( E[x^2] \) is expected value of the squared Rayleigh distributed envelope. Since \( x(t) \) is Rayleigh distributed, the instantaneous received SNR \( \gamma_b = x^2(t)/N_0 \) is Chi-square distributed with 2 degrees of freedom [35]. The chi-square probability density function with 2 degrees of freedom is given by

\[
f_{Y_b}(\gamma_b) = \frac{1}{\gamma_b^{\frac{3}{2}}} e^{-\gamma_b/\gamma_b}, \quad \gamma_b \geq 0
\]  

(2.72)

The probability of error for a Rayleigh fading signal can be determined by averaging the probability of error for AWGN channel given by Eq.(2.70) over the PDF of \( \gamma_b \). Therefore,

\[
P_e = \int_0^\infty Q\left(\sqrt{2\gamma_b}\right) f_{Y_b}(\gamma_b) \, d\gamma_b
\]  

(2.73)

Substituting Eq.(2.72) in eq.(2.73) and solving we have

\[
P_e = \frac{1}{2} \left(1 - \sqrt{\frac{\gamma_b}{1 + \gamma_b}}\right)
\]  

(2.74)

Fig. 2.4 compares the average probability of error for fading channels with AWGN

![Figure 2.4: \( P_e \) for AWGN channel and Rayleigh fading channel](image)

only channels. The probability of error for AWGN channel is determined using
Eq.(2.70) and Rayleigh fading channels using Eq.(2.74). Error probabilities are plotted on a log scale along the Y-axis. To achieve average bit error probability of $10^{-2}$, the AWGN channel requires SNR of 4 dB. The SNR requirement increases by a factor of 3 for fading channels for the same probability of error.

For BPSK modulation, the average probability of error for Rayleigh fading signals is also given by the shaded area in Fig. 2.3. This area is given by the integral

$$ P_e = \int_{0}^{\infty} f_R(r \mid \varphi) \, dr $$

(2.75)

where $f_R(r \mid \varphi)$ is given by Eq.(2.58). Fig. 2.5 compares the error probability obtained by evaluating Eq.(2.75) using numerical integration. The curves match closely.

![Figure 2.5: Probability of error using theoretical vs numerical integration method](image)

### 2.7.3 Amplitude values causing fade

In this section, a threshold based error analysis is performed. $\chi_{TH}$ is the threshold value on the Rayleigh distributed fading envelope $x(t)$ such that the probability
of error $P_e^{TH}$ caused by $0 \leq x(t) < \chi_{TH}$ is related to the average probability of error $P_e$ as

$$P_e^{TH} \leq C \cdot P_e, \quad 0 < C < 1$$

(2.76)

$P_e$ is given by Eq.(2.74). $C$ is a constant factor and equals the fraction of total errors that are caused by the amplitude range $[0 : \chi_{TH}]$. $C = 1$ corresponds to the range of amplitudes $[0, \infty]$. From Fig. 2.6, the following relationship can be established

$$\frac{P_e^{TH}}{P_e} = \frac{\int_{0}^{\chi_{TH}} f_R(r | \varphi) \, dr}{\int_{0}^{\infty} f_R(r | \varphi) \, dr}$$

(2.77)

Eq.(2.77) can be solved numerically to yield the value $\chi_{TH}$.

![Figure 2.6: Received signal PDF $f_R(r | \varphi)$ and $\chi_{TH}$](image)

Error thresholds on the envelope $x(t)$ and the inphase component $y(t)$ and the quadrature component $z(t)$ can be obtained by taking the expectation of Eq.(2.57), given $\varphi(t)$ and $r(t) = \chi_{TH}$ as

$$\chi_{TH} = e^{j \varphi(t)} E[x(t)] + E[\eta(t)]$$

(2.78)

$$= e^{j \varphi(t)} E[x(t)]$$
Since the noise is a zero mean process, the second term in the above sum is zero. Therefore the expected value of \( x(t) \) that causes \( C\% \) of errors is proportional to the expected value of \( r(t) \) contributing to the same fraction of errors. The threshold on the inphase/quadrature components can be obtained by taking the expectation of Eq.(2.43) as

\[
E\left[ x(t) \right] = E\left[ \sqrt{y^2(t) + z^2(t)} \right]
\]

(2.79)

When \( C\% \) of errors are considered, \( E\left[ x(t) \right] = \chi_{TH} \) using Eq.(2.78). The maximum value of \( y^2(t) \) that contributes to \( C\% \) of errors occurs when \( z^2(t) = 0 \). Solving Eq.(2.79) gives

\[
-\chi_{TH} \leq E[y(t)] \leq \chi_{TH}
\]

(2.80)

Similarly,

\[
-\chi_{TH} \leq E[z(t)] \leq \chi_{TH}
\]

(2.81)

![Figure 2.7: Transmitted SNR (dB) vs \( \chi_{TH} \)]
Fig. 2.7 depicts variation in the value of $\chi_{TH}$ with transmitted SNR $\frac{2N_0}{N_0}$. $\chi_{TH}$ is evaluated using Eq. (2.77) through numerical integration. Since a closed form solution does not exist for $\frac{N_0}{2} \neq 1$, the desired PDF $f_R(r \mid \varphi)$ is evaluated from Eq. (2.58) using numerical integration. The parameters used for $f_R(r \mid \varphi)$ are $\Omega_p = 1$ and $\frac{N_0}{2}$ corresponding to $\epsilon_b = 1$. As SNR increases, the range of amplitudes contributing to error reduce thereby reducing the threshold.

2.8 Summary

In this chapter, the statistical characteristics of Rayleigh flat fading channel are analyzed. In particular, the PDF of the received signal $f(r \mid \varphi)$ is derived under coherent detection conditions and a threshold $\chi_{TH}$ on the received signal is identified. When the received signal $|r(t)|$ falls in the range $[0 : \chi_{TH}]$, $C\%$ of total errors occur. Though this range is specific to BPSK modulation scheme, it can also be extended to other modulation schemes. The threshold on $r(t)$ is translated into the corresponding value for the received envelope $x(t)$ and its inphase and quadrature components $y(t)$ and $z(t)$ respectively. When the envelope falls in the range $0 \leq E[x(t)] \leq \chi_{TH}$ or its inphase and quadrature components are in the range $-\chi_{TH} \leq E[y(t)], E[z(t)] \leq \chi_{TH}$, $C\%$ of the total errors are caused. Hence, $\chi_{TH}$ partitions the fading channel into error and non-error states.

As discussed in this chapter, the wireless Rayleigh flat fading channel is intrinsically correlated and therefore predictable. The prediction should ideally give an "ahead of time" estimate of the channel to the error states so that proactive corrective measures can be initiated. In the next chapter, dynamic models are presented
that capture the transitions of the fading channel in and out of error and/or non-error states. It will be shown that the models using first order memory are inadequate and at-least a second order memory is required to represent the fading channels features on short time scales.
CHAPTER 3

STOCHASTIC MODELS FOR FAIDING CHANNELS

3.1 Introduction

Stochastic models of wireless channels attempt to capture the temporal dynamics of the channel. This is particularly important for multipath channels that exhibit significant correlation in time. Knowledge of the channel state may allow one to control transmission rates and minimize errors.

Stochastic channel models such as the Markov chain and those based on more general time-series models have been applied to model channel time variations [30][31] [32][33]. In this chapter, Markov chain models and autoregressive processes of first and second order are applied to model the Rayleigh fading channels. The model performance is evaluated for short and long observation durations.

3.2 Markov Chain Model

A 2-state Markov chain (MC) is proposed as a model for the Rayleigh distributed signal envelope $x(t)$. The signal $x(t)$ is sampled at discrete time intervals spaced $\Delta t$ apart. The samples are denoted as

$$x_n = x(n\Delta t) \quad (3.1)$$
An observation vector $\underline{X}$ is represented as

$$\underline{X} = \begin{bmatrix} x_n \\ x_{n-1} \end{bmatrix}$$ (3.2)

To map the channel variations into two states, the PDF $f_X(x)$ is partitioned into two states labeled $s_1$ and $s_2$, using the threshold value $\chi_{TH}$ derived in Section 2.7.3. The threshold is selected such the ratio $P_{eTH}/P_e = 0.9999$. The two state process is

$$x_n = \begin{cases} 
  s_1 & \text{if } 0 \leq x_n < \chi_{TH} \\
  s_2 & \text{if } \chi_{TH} \leq x_n < \infty
\end{cases}$$ (3.3)

Thus, the amplitudes of the signal $x(t)$ are partitioned into two regimes yielding two possible output symbols $s_1$ and $s_2$. Note that these symbols are not the same $s_1$, $s_2$ used for BPSK in Eq. (2.66). The state of the MC as a function of time is given by $S_n$. The transition probability matrix for the MC is defined by a 2x2 matrix

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$ (3.4)

where, $p_{ij} = Pr[S_n = s_j \mid S_{n-1} = s_i], \quad i, j = 1, 2; \quad n = 0, 1, 2, \cdots$

The transition probabilities $p_{ij}$ are defined as

$$p_{ij} = Pr[S_n = s_j \mid S_{n-1} = s_i] = \frac{Pr[S_n = s_j, S_{n-1} = s_i]}{Pr[S_{n-1} = s_i]}$$ (3.5)

where $Pr[.]$ denotes the probability of the argument. Under steady-state conditions, for a fixed value of $\chi_{TH}$, the denominator in Eq. (3.5) can be evaluated for the Rayleigh distribution $f_X(x)$ as

$$Pr[S_n = s_i] = \begin{cases} 
  \frac{\chi_{TH}}{0} f_X(x) \, dx, \quad i = 1 \\
  \frac{\chi_{TH}}{\infty} f_X(x) \, dx, \quad i = 2
\end{cases}$$ (3.6)
The numerator in Eq.(3.5) can be obtained by integrating the joint PDF $f_X(x_n, x_{n-1})$

$$Pr[S_n = s_j, S_{n-1} = s_i] = \int \int f_X(x_n, x_{n-1}) \, dx_n \, dx_{n-1} \quad (3.7)$$

The joint PDF for Rayleigh distributed variables will be derived in the next section.

In the limit as $n \to \infty$, the probabilities of the process occupying each state is given by the steady state probability vector

$$\Pi = [\pi_1 \, \pi_2] \quad (3.8)$$

Applying the Chapman-Kolmogorov equation [60], as $n \to \infty$ leads to the following relationship between $\Pi$ and $P$

$$\Pi P = \Pi \quad (3.9)$$

Substituting $P$ from Eq.(3.4) and solving for $\pi_1$ and $\pi_2$ under the condition $\pi_1 + \pi_2 = 1$, it can be shown that

$$\pi_1 = \frac{p_{21}}{p_{12} + p_{21}} \quad (3.10)$$

$$\pi_2 = \frac{p_{12}}{p_{12} + p_{21}}$$

To find the values of $s_1$ and $s_2$, it is first assumed that $s_1$ is the mean amplitude in $[0, \chi_{TH}]$ and therefore derived as

$$s_1 = \frac{\int \chi_{TH} x f_X(x) \, dx}{\int f_X(x) \, dx} \quad (3.11)$$

Once $s_1$ is obtained, the value of $s_2$ can be determined by matching the first moment of the Markov process to that of the Rayleigh distribution. The expected value of the Rayleigh PDF $x_{\text{mean}}$ is given by Eq.(2.46). The first moment of Markov chain is

$$x_{\text{mean}} = s_1 \pi_1 + s_2 \pi_2 \quad (3.12)$$
Equating $x_{\text{mean}}$ and $s_{\text{mean}}$ and solving for $s_2$ yields

$$s_2 = \frac{x_{\text{mean}} - \pi_1 s_1}{\pi_2}$$  \quad (3.13)

### 3.3 Joint Probability Density Function: Rayleigh Channel

The two dimensional joint PDF for the Rayleigh distributed envelope is derived in this section. The solution provides the numerator in Eq.(3.5). The Gaussian distributed inphase component $y(t)$ and quadrature component $z(t)$ are considered in the derivation. The samples are represented as $y_n$ and $z_n$

$$y_n = y(n\Delta t), \quad z_n = z(n\Delta t)$$  \quad (3.14)

and the phase

$$\phi_n = \tan^{-1} \frac{z_n}{y_n}$$  \quad (3.15)

Therefore,

$$x_n = \sqrt{y_n^2 + z_n^2}$$  \quad (3.16)

$$y_n = x_n \cos \phi_n, \quad z_n = x_n \sin \phi_n$$  \quad (3.17)

The observations are aggregated in vectors $\Phi$, $\mathbf{Y}$ and $\mathbf{Z}$ where,

$$\Phi = \begin{bmatrix} \phi_n & \phi_{n-1} \end{bmatrix}$$  \quad (3.18)

$$\mathbf{Y} = \begin{bmatrix} y_n & y_{n-1} \end{bmatrix}$$

$$\mathbf{Z} = \begin{bmatrix} z_n & z_{n-1} \end{bmatrix}$$
The joint PDF \( f_X(x_n, x_{n-1}) \) is evaluated by constructing the function \( f_{X\Phi}(x_n, \phi_n, x_{n-1}, \phi_{n-1}) \), using the transformations given by Eq. (3.17). The marginal PDF \( f_X(x_n, x_{n-1}) \) is then derived as

\[
f_X(x_n, x_{n-1}) = \int_0^{2\pi} \int_0^{2\pi} f_{X\Phi}(x_n, \phi_n, x_{n-1}, \phi_{n-1}) \, d\phi_n \, d\phi_{n-1}
\tag{3.19}
\]

assuming \( \phi_n \) is uniformly distributed in \([0 : 2\pi]\). The joint PDF \( f_{X\Phi}(x_n, \phi_n, x_{n-1}, \phi_{n-1}) \) can be determined from the function \( f_{YZ}(y_n, z_n, y_{n-1}, z_{n-1}) \) through the following transformation [57]

\[
f_{YZ}(y_n, z_n, y_{n-1}, z_{n-1}) = |J| \, f_{X\Phi}(x_n, \phi_n, x_{n-1}, \phi_{n-1})
\tag{3.20}
\]

where \( J \) is the Jacobian of the transformation defined as

\[
J = \begin{bmatrix}
\frac{\partial y_n}{\partial x_n} & \frac{\partial y_n}{\partial \phi_n} & \frac{\partial y_n}{\partial x_{n-1}} & \frac{\partial y_n}{\partial \phi_{n-1}} \\
\frac{\partial z_n}{\partial x_n} & \frac{\partial z_n}{\partial \phi_n} & \frac{\partial z_n}{\partial x_{n-1}} & \frac{\partial z_n}{\partial \phi_{n-1}} \\
\frac{\partial y_{n-1}}{\partial x_n} & \frac{\partial y_{n-1}}{\partial \phi_n} & \frac{\partial y_{n-1}}{\partial x_{n-1}} & \frac{\partial y_{n-1}}{\partial \phi_{n-1}} \\
\frac{\partial z_{n-1}}{\partial x_n} & \frac{\partial z_{n-1}}{\partial \phi_n} & \frac{\partial z_{n-1}}{\partial x_{n-1}} & \frac{\partial z_{n-1}}{\partial \phi_{n-1}}
\end{bmatrix}
\tag{3.21}
\]

Using the relationships in Eq. (3.17), the determinant \(|J|\) is obtained as

\[
|J| = x_n \, x_{n-1}
\tag{3.22}
\]

The random variates \( y_n \) and \( z_n \) are independent and Gaussian distributed as \( N(0, \frac{\sigma_m^2}{2}) \). The derivation of the multivariate Gaussian PDF is given first. The joint multivariate Gaussian PDF is used to obtain \( f_{YZ}(y_n, z_n, y_{n-1}, z_{n-1}) \) in Eq. (3.20).

The joint PDF of a set of Gaussian variables \( \mathbf{W} = [W_1 \, W_2 \, \cdots \, W_m] \) with individual distributions \( N(\mu_m, \sigma_m^2), m = 1, 2, \cdots, M \) is given by [56]

\[
f_W(w_1, \ldots, w_M) = \exp \left[ -\frac{1}{2\lambda^2} \sum_{m=1}^{M} \sum_{k=1}^{K} |\Lambda|_{mk} w_m w_k \right] \\
\frac{1}{(2\pi)^{M/2} |\Lambda|^{1/2}}
\tag{3.23}
\]
where $|\Lambda|$ is the determinant of the covariance matrix

$$
\Lambda = \begin{vmatrix}
\lambda_{11} & \lambda_{12} & \cdots & \lambda_{1M} \\
\lambda_{21} & \lambda_{22} & \cdots & \lambda_{2M} \\
\vdots & \vdots & \ddots & \vdots \\
\lambda_{M1} & \lambda_{M2} & \cdots & \lambda_{MM}
\end{vmatrix}
$$

(3.24)

and $|\Lambda|_{nm}$ is the cofactor of the element $\lambda_{nm}$. The elements are

$$
\lambda_{mk} = E \left[ (w_m - \mu_m)(w_k - \mu_k) \right]
$$

(3.25)

The joint PDF $f_{YZ}(y_n, z_n, y_{n-1}, z_{n-1})$ is obtained by using $M = 4$ in Eq.(3.23) such that

$$
w_1 = y_n \quad w_2 = z_n
$$

$$
w_3 = y_{n-1} \quad w_4 = z_{n-1}
$$

The determinant

$$
|\Lambda| = \begin{vmatrix}
E(y_n y_n) & E(y_n z_n) & E(y_n y_{n-1}) & E(y_n z_{n-1}) \\
E(z_n y_n) & E(z_n z_n) & E(z_n y_{n-1}) & E(z_n z_{n-1}) \\
E(y_{n-1} y_n) & E(y_{n-1} z_n) & E(y_{n-1} y_{n-1}) & E(y_{n-1} z_{n-1}) \\
E(z_{n-1} y_n) & E(z_{n-1} z_n) & E(z_{n-1} y_{n-1}) & E(z_{n-1} z_{n-1})
\end{vmatrix}
$$

(3.27)

For the case of zero mean processes $y_n$ and $z_n$

$$
E(y_{n-1} y_n) = \rho_{yy}(\Delta t)
$$

$$
E(z_{n-1} z_n) = \rho_{zz}(\Delta t)
$$

$$
E(y_{n-1} z_n) = -E(z_{n-1} y_n) = \rho_{yz}(\Delta t)
$$
Since inphase and quadrature components are independent the cross-correlation terms in $\Lambda$ are zero. From Eq. (2.39) $\rho_{yy}(\Delta t) = \rho_{zz}(\Delta t)$ and $\sigma_y^2 = \sigma_z^2 = \frac{\Omega_p}{2}$. Substituting Eqs. (3.28) in (3.27) the covariance matrix can be rewritten as

$$\Lambda = \begin{bmatrix}
\frac{\Omega_p}{2} & 0 & \rho_{yy}(\Delta t) & 0 \\
0 & \frac{\Omega_p}{2} & 0 & \rho_{zz}(\Delta t) \\
\rho_{yy}(\Delta t) & 0 & \frac{\Omega_p}{2} & 0 \\
0 & \rho_{zz}(\Delta t) & 0 & \frac{\Omega_p}{2}
\end{bmatrix}$$  

(3.29)

The joint PDF $f_{y,z}(y_n, z_n, y_{n-1}, z_{n-1})$ can be determined by substituting Eq. (3.26) and Eq. (3.29) in Eq. (3.23). Substituting the result in Eq. (3.20) gives $f_{X \Phi}(x_n, \phi_n, x_{n-1}, \phi_{n-1})$. Evaluating the marginal PDF using Eq. (3.19), the final expression for the joint PDF can be reduced to [56]

$$f_X(x_n, x_{n-1}) = \frac{x_{n-1}x_n}{|\Lambda|^{1/2}} I_0(q) \exp\left(-\frac{\rho_{yy}(\Delta t)}{4|x_n|^2}\right), \quad x_n, x_{n-1} \geq 0$$

$$= 0 \quad \text{otherwise}$$

(3.30)

where $q = \frac{x_{n-1}x_n}{|\Lambda|^{1/2}} \sqrt{\frac{\rho_{yy}(\Delta t)}{4|x_n|^2}}$

where $I_0(q)$ is the modified Bessel function of zeroth order. Substituting the above result in Eq. (3.7) gives the Markov transition probability in Eq. (3.5).

The autoregressive (AR) process is considered next. The AR models are used to independently model the inphase and the quadrature components. In order to obtain the AR parameters, the statistical analysis of Gaussian distributed inphase and quadrature components is performed first.
3.4 Multivariate Gaussian Process Analysis

The PDF of $y_n : N(0,1)$ is

$$f_Y(y_n) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y_n^2}{2}} \quad (3.31)$$

From the law of conditional probability [56]

$$f_Y(y_M|y_1,\ldots,y_{M-1}) = \frac{f_Y(y_1,\ldots,y_{M-1},y_M)}{f_Y(y_1,\ldots,y_{M-1})} \quad (3.32)$$

and Eq.(3.23), the joint PDF can be obtained as

$$f_Y(y_n, y_{n-1}) = \frac{1}{2\pi\sqrt{1-\rho^2}} e^{-\frac{y_n^2 - 2\rho y_n y_{n-1} + y_{n-1}^2}{1-\rho^2}} \quad (3.33)$$

where,

$$\rho_k = \rho_{yy}(k\Delta t_s) \quad (3.34)$$

and $\Delta t_s$ is the symbol interval. The PDF of $y_n$ conditioned on $y_{n-1}$ is obtained by dividing Eq.(3.33) by Eq.(3.31) to get

$$f_Y(y_n|y_{n-1}) = \frac{1}{\sqrt{2\pi(1-\rho^2)}} e^{-\frac{(y_n - \rho y_{n-1})^2}{2(1-\rho^2)}} \quad (3.35)$$

It follows from Eq.(3.35) that the conditional PDF $f_Y(y_n|y_{n-1})$ is Gaussian with mean

$$E[y_n|y_{n-1}] = \rho y_{n-1} \quad (3.36)$$

and variance

$$Var[y_n|y_{n-1}] = (1 - \rho^2) \quad (3.37)$$
Similarly, the joint PDF of $y_n, y_{n-1}, y_{n-2}$ can be obtained as

$$f_Y(y_n, y_{n-1}, y_{n-2}) = \frac{1}{\sqrt{(2\pi)^3\sigma^2}} e^{-\frac{\xi^2}{2\sigma^2}}$$ \hspace{1cm} (3.38)

where, \(\sigma^2 = 1 - \rho_2^2 + 2\rho_1\rho_2 - 2\rho_1^2\)

\[\xi = (1 - \rho_1^2)(y_{n-2}^2 + y_n^2) + (1 - \rho_2^2)y_{n-1}^2 + 2\rho_1(\rho_2 - 1)(y_{n-2}y_{n-1} + y_{n-1}y_n) + 2(\rho_1^2 - \rho_2^2)y_{n-2}y_n\]

and the PDF of $y_n$ conditioned on $y_{n-1}, y_{n-2}$

$$f_Y(y_n | y_{n-1}, y_{n-2}) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\psi^2}{2\sigma^2}}$$ \hspace{1cm} (3.39)

where \(\sigma^2 = \frac{(-\rho_2^2 + 2\rho_1\rho_2 - 2\rho_1^2 + 1)}{(1 - \rho_1^2)}\)

\[\psi = y_{n-1}\frac{\rho_1(1 - \rho_2)}{(1 - \rho_1^2)} + y_{n-2}\frac{(\rho_2 - \rho_1^2)}{(1 - \rho_1^2)}\]

It follows from Eq.(3.39) that the second order process is also Gaussian distributed with expected value

$$E[y_n | y_{n-1}, y_{n-2}] = y_{n-1}\frac{\rho_1(1 - \rho_2)}{(1 - \rho_1^2)} + y_{n-2}\frac{(\rho_2 - \rho_1^2)}{(1 - \rho_1^2)}$$ \hspace{1cm} (3.40)

and variance

$$Var[y_n | y_{n-1}, y_{n-2}] = \frac{(1 - \rho_2^2 + 2\rho_1^2\rho_2 - 2\rho_1^2)}{(1 - \rho_1^2)}$$ \hspace{1cm} (3.41)

The statistics for the quadrature component $z_n$ are identical to that of $y_n$.

### 3.5 Autoregressive Models

In this section a time-series model is proposed for the Rayleigh fading channel.

The inphase and quadrature components are separately modeled as independent AR processes. Since the inphase and quadrature components have the same variance $\frac{\sigma^2}{2}$,
and ACF $\frac{\omega_m}{2} J_0(\omega_m \Delta t)$, the same set of AR parameters can be used to characterize both processes. The estimated Rayleigh envelope $x_n$ is obtained from Eq.(3.16) using the models for the inphase and quadrature components.

An AR process results from the integration of the weighted Gaussian random variates and can be represented as

$$y_n = \eta_n + \sum_{j=1}^{\infty} \psi_j \eta_{n-j}$$

(3.42)

where $\eta_n$ is a Gaussian white noise process $N(0, \sigma^2_N)$ and $\psi_j$ is the weighting coefficient. The infinite sum in the above equation can be rearranged as a combination of an autoregressive and a noise component such that, the $p^{th}$ order AR process $AR - p$ is

$$y_n = \sum_{j=1}^{p} \Phi_j y_{n-j} + \eta_n$$

(3.43)

where $\Phi_j$ is the regression coefficient.

### 3.6 Model Selection

The current work in wireless systems and networks [10]-[22] consider first order Markov models. While the first order model can accurately characterize the long time-averaged channel statistics such as average error probability and average level crossing rate, the channel temporal dynamics on smaller time-scales, particularly at the packet level are not well modeled. The accuracy of first order approximation of fading channels is investigated in [34] and inadequacies are identified. Higher order AR models have been used in [30]-[33]. Howard et al. have found the second order model adequate for Rayleigh fading channels. In this section, the AR model selection
based on the ACF is discussed.

For a flat Rayleigh fading channel, the ACF of inphase and quadrature components is given by the Bessel function given in Eq. (2.38)

\[ \rho_{yy}(\Delta t) = \frac{\Omega_p}{2} J_0(\omega_m \Delta t) \quad (3.44) \]

where \( \omega_m = 2\pi f_m \) and \( f_m \) is the Doppler frequency. The Bessel function \( J_0(z) \) can be expanded as an infinite sum in \( z^2 \) as \([58]\)

\[ J_0(z) = 1 - \frac{\frac{z^2}{(1!)^2}}{1} + \frac{\left(\frac{z^2}{2!}\right)^2}{2!} - \frac{\left(\frac{z^2}{3!}\right)^3}{3!} + \cdots \quad (3.45) \]

where \( z = \omega_m \Delta t \). In the limit as \( \Delta t \rightarrow 0 \) Eq. (3.45) can be approximated as

\[ \hat{\rho}_{yy}(\Delta t) = \lim_{\Delta t \rightarrow 0} \rho_{yy}(\Delta t) \approx \frac{\Omega_p}{2} \left[ 1 - \left(\frac{\omega_m \Delta t}{2}\right)^2 \right] + O\left(\left(\frac{\omega_m \Delta t}{2}\right)^4\right) \quad (3.46) \]

where the argument of \( O(.) \) represents the tail of the series. Hence the ACF can be modeled by a parabolic function having symmetry about \( \Delta t = 0 \). The first zero of \( \hat{\rho}_{yy}(\Delta t) \) occurs at \( \omega_m \Delta t = 2 \) whereas \( \rho_{yy}(\Delta t) = 0 \) occurs at \( \omega_m \Delta t = 2.404 \). One can therefore expect that a low order time-series can model the correlation in the regime \( 0 \leq \omega_m \Delta t < 2 \). The first order approximation of the Bessel function can be matched by a function of the form

\[ \frac{\Omega_p}{2} \cos(\omega_m \Delta t) = \frac{\Omega_p}{2} \sum_{k=0}^{\infty} (-1)^k \frac{(\omega_m \Delta t)^k}{(2k)!} \quad (3.47) \]

which, in the limit as \( \Delta t = 0 \) is

\[ \frac{\Omega_p}{2} \cos(\omega_m \Delta t) = \frac{\Omega_p}{2} \left[ 1 - \left(\frac{\omega_m \Delta t}{2}\right)^2 \right] \quad (3.48) \]

This gives a reason to believe that a second order AR model may be effective in modeling the amplitude process.
In this work, first and second order models are examined. The PDF of $y_n$ conditioned on $y_{n-1}$ for first order and conditioned on $y_{n-1}$ and $y_{n-2}$ for second order process are used to obtain the AR parameters. The conditional distributions are derived in Section 3.4.

### 3.6.1 First Order AR Model

Using the first order autoregressive process (AR-1), the inphase component $y_n$ is given by:

$$y_n = \Phi_1 y_{n-1} + \eta_n$$  \hspace{1cm} (3.49)

The objective is to find the regression parameter $\Phi_1$ and the noise variance $\sigma^2_N$. Using the above equation and $E[\eta_n] = 0$, the conditional expectation can be written as

$$E[y_n | y_{n-1}] = \Phi_1 y_{n-1}$$  \hspace{1cm} (3.50)

and the second moment can be written as

$$E[y_n^2 | y_{n-1}] = (\Phi_1 y_{n-1})^2 + E[\eta^2_n] + 2 \Phi_1 y_{n-1} E[\eta_n]$$  \hspace{1cm} (3.51)

Since $\eta_n$ and $y_n$ are uncorrelated processes, the conditional variance is

$$Var[y_n | y_{n-1}] = E[y_n^2 | y_{n-1}] - (E[y_n | y_{n-1}])^2$$  \hspace{1cm} (3.52)

$$= \sigma^2_N$$

Comparing Eq. (3.36) and (3.50), the value of $\Phi_1$ can be written as

$$\Phi_1 = \rho_1$$  \hspace{1cm} (3.53)
The variance of \( y_n : N(0, 1) \) conditioned on \( y_{n-1} \) is given by Eq.(3.37). Comparing Eq.(3.37) and (3.52) and considering \( y_n : N(0, \frac{\Omega}{2}) \), the AR-1 noise variance can be obtained as

\[
\sigma_N^2 = \frac{\Omega}{2} (1 - \rho_1^2)
\]  

(3.54)

For an AR-1 process, the ACF is is given by [61]

\[
E[y_n y_{n+k}] = \rho_k = \Phi_k^k
\]  

(3.55)

The spectrum \( S(f) \) for the AR-1 process correspondingly is [61]

\[
S(f) = \frac{2\sigma_N^2}{|1 - \Phi_1 e^{-j2\pi f}|^2} \quad 0 \leq f \leq \frac{1}{2}
\]  

(3.56)

### 3.6.2 Second Order AR Model

The second order autoregressive process (AR-2) is given by

\[
y_n = \Phi_1 y_{n-1} + \Phi_2 y_{n-2} + \eta_n
\]  

(3.57)

The conditional statistics in terms of regression parameters are

\[
E[y_n | y_{n-1}, y_{n-2}] = \Phi_1 y_{n-1} + \Phi_2 y_{n-2}
\]  

(3.58)

\[
Var[y_n | y_{n-1}, y_{n-2}] = \sigma_N^2
\]  

(3.59)

Comparing Eq.(3.58) with Eq.(3.40) the regression coefficients can be obtained as

\[
\Phi_1 = \frac{\rho_1 (1 - \rho_2)}{1 - \rho_1^2}
\]  

(3.60)

\[
\Phi_2 = \frac{\rho_2 - \rho_1^2}{1 - \rho_1^2}
\]
Comparing Eq. (3.59) with Eq. (3.41), the noise variance of AR-2 process when $y_n : N(0, \frac{\Omega}{4})$ is

$$\sigma_N^2 = \frac{\Omega \rho_0}{2} \frac{(1 - \rho_1^2 + 2\rho_0^2 - 2\rho_1^2)}{(1 - \rho_1^2)}$$

$$= \frac{\Omega \rho_0}{2} \left(1 - \Phi_1 \rho_1 - \Phi_2 \rho_2\right)$$

(3.61)

To compute stability requirements on the values of regression coefficients the constraints $1 - \rho_1^2 > 0$ and $Var(y_n | y_{n-1}, y_{n-2}) > 0$ are applied. These conditions translate into the following constraints on the AR coefficients

$$\Phi_1 + \Phi_2 < 1$$

(3.62)

$$\Phi_2 - \Phi_1 < 1$$

$$-1 < \Phi_2 < 1$$

The normalized autocorrelation coefficient for the AR-2 model for the $k^{th}$ lag is [61]

$$\rho_k = \Phi_1 \rho_{k-1} + \Phi_2 \rho_{k-2}, \quad k > 1$$

(3.63)

with starting values $\rho_0 = 1$ and $\rho_1 = \Phi_1/(1 - \Phi_1)$. The spectral density for the AR-2 model is given by [61]

$$S(f) = \frac{2\sigma_N^2}{|1 - \Phi_1 e^{-i2\pi f} - \Phi_2 e^{-i4\pi f}|^2} \quad 0 \leq f \leq \frac{1}{2}$$

(3.64)

### 3.7 Model Performance

The performance characteristics of Rayleigh fading channels are compared with the models given by the Markov chain and the AR processes. For Rayleigh fading
channels, the received signal is \( r_n = x_n e^{j \varphi_n(t)} + \eta_n \) where \( x_n \) is Rayleigh distributed and \( \varphi_n(t) \) is known at the receiver under the assumption of coherent detection. The value of \( x_n \) is generated using the MC and AR models. The MC parameters are obtained for each threshold \( \chi_{TH} \) derived as a function of SNR value. The parameters for the AR processes are given by Eq.(3.53) for the first order and by Eq.(3.60) for the second order process. Correlation coefficients required for evaluation of AR parameters are evaluated at the symbol interval of \( \Delta t_s = 72 \times 10^{-6} \) for Doppler frequency of 90 Hz and \( \Omega_p = 1 \). The estimates for inphase and quadrature components using AR models are combined using Eq.(3.16) to obtain the Rayleigh distributed envelope. A million symbols are considered in the simulation for each case.

![Figure 3.1: Average probability of bit errors](image)

3.7.1 Comparison of Average Bit Error Probability

A comparison of the average probability of bit errors is presented in this section. The fading AWGN channel is simulated for SNR in the range \([2 : 25] \) dB. The modulation scheme is binary phase shift keying (BPSK). The average probability
of bit errors is evaluated for Markov chain, AR-1 and AR-2 models and plotted in Fig. 3.1 against the theoretical values given by Eq.(2.73). The error probabilities generated by the models closely follow the theoretical values, with some deviation for the MC at SNR \( \leq 10 \) dB.

3.7.2 Comparison of Block Errors

The models are also evaluated with respect to block-errors. The simulation based analysis is considered in this section. The numerical approach for the problem is presented in the next chapter. The block error statistics obtained from \( x_n \) generated by the models are compared with those generated using the Jake’s simulator. The Jake’s simulator is described in Appendix-B. A channel with SNR=10 dB is simulated. BPSK modulation is used and blocks of \( n = 10 \) to 100 bits are considered. A block is assumed in error if at-least one bit in the block is detected in error. The average block error probability is calculated as the ratio of the total number of blocks received in error to the total number of transmitted blocks. Fig.
3.2 shows the average probability of block errors for each model. The MC generates significant over-estimation of the block error probability for all the block sizes. The AR models exhibit closer agreement with the Rayleigh trace for block-size up to 20 blocks. AR-1 process underestimates the probabilities for block sizes greater than 20. AR-2 provides good estimates for the entire range of block sizes considered.

![Graphs showing cumulative distribution function for block errors using simulation](image)

(a) Block Size 10 bits  
(b) Block Size 100 bits

Figure 3.3: Cumulative distribution function for block errors using simulation

In Fig. 3.3 the cumulative probability distribution function (CDF) $P(M \leq m)$ for $m$ errors in a block size of $n$ bits is plotted. A block size of 10 bits is used in (a) and 100 bits in (b). The model and the simulated statistics are in close agreement when block size is 10 bits. For block size of 100 bits, while the CDF obtained using the AR-2 model matches with that for the simulated Rayleigh trace. On the other hand, the first order model exhibits much higher probability for blocks with no errors. The disagreement exists due to underlying differences in the dynamics of the error process generated by AR-1 and AR-2 models. These dynamics are examined next for each case.
3.7.3  Comparison of Autocorrelation Function

The ability of models in capturing second order statistics is examined in this section. The ACF for inphase and quadrature components is given by the Bessel function \( \frac{\Omega_0}{2} J_0(2\pi f_m \Delta t) \) as shown in Eq. (2.38). For the Bessel function plot, \( \Delta t = n\Delta t_s, n = 0, 1, 2 \cdots \) is used. The normalized ACF for AR-1 and AR-2 models is evaluated using Eq. (3.55) and Eq. (3.63) respectively. For the MC model, the normalized ACF is evaluated from simulated values. Fig. 3.4 shows that the MC and AR-1

![Bessel Function](image)

Figure 3.4: Comparison of Autocorrelation function for AR-1 and AR-2 processes with Bessel function

match with the Bessel function correlation for lags \( n \leq 2 \) matching only the variance (n=0) and the first lag (n=1). The AR-2 performs significantly better, matching up to 50 lags. The close approximation by the AR-2 process is due to the oscillating trend the AR-2 is capable of exhibiting. The envelope that damps the oscillations of the Bessel function is however not captured by AR-2 model. The block-error process for Rayleigh fading channels is poorly modeled by MC and AR-1 model due to incorrect ACF structure. The effect of poorly estimated ACF on the error process is
examined next.

### 3.7.4 Comparison of Level Crossings

The level crossing statistics for Rayleigh fading are calculated using Eq.(2.48) and the level crossings for the AR processes are obtained using simulation. The inphase and quadrature components are simulated using independent AR processes and the result is combined using Eq.(3.16) to obtain the signal. Since level crossings are relevant only for continuous processes, Markov chains are not considered in this analysis.

![Rayleigh Envelope](image)

Figure 3.5: Average level crossing rate

In Fig. 3.5, the average number of level crossings is examined for the level $R$ ranging from -30dB to 10dB. The values obtained from Eq.(2.48) are shown as a solid line. AR-2 performs uniformly well. The AR-1 slightly underestimates the theoretical values at low values of $R$.

The trend of level crossings in time is examined next using simulation. A level $R = 0$ is chosen. A million samples are used to obtain the statistic. Table
3.1 shows the minimum, maximum and the average number of samples between the level crossings using AR-1 and AR-2 models. Comparing these values with those obtained using the Jake’s simulator, it is clear that the variability in AR-1 process is much higher than that for the AR-2 process. Higher variability is also observable in Fig. 3.6(a) where the cumulative count of level-crossings in time are shown. Fig. 3.6(b) depicts a closer look at dynamics for the first 5000 to 20000 points. The level crossings do not ensue until 9500 points in AR-1 model. The AR-1 process exhibits sudden jumps whereas the AR-2 has uniformly distributed level-crossings in time. Examination of level-crossing dynamics explains the disagreement in block error statistics shown in Fig. 3.3. In a fixed time interval, the first order model generates a large number of error clusters in succession followed by a long error-free run. In contrast, the second order model distributes errors uniformly across the time. When block-sizes are smaller than the size of error clusters, this difference does not result in significant disagreement between AR-1 and AR-2 statistics. When
<table>
<thead>
<tr>
<th></th>
<th>Inphase Components (Jake’s)</th>
<th>AR-1</th>
<th>AR-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>2</td>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td>Maximum</td>
<td>776</td>
<td>17360</td>
<td>424</td>
</tr>
<tr>
<td>Average</td>
<td>174</td>
<td>252</td>
<td>218</td>
</tr>
</tbody>
</table>

Table 3.1: Number of samples between zero crossings of inphase component

The block-size is increased, the AR-1 model results in large number of bits in error in one block followed by a succession of error free blocks resulting in high probability of blocks with no error. On the other hand, the AR-2 model results in a large number of blocks in error, with fewer bits in error in each, unlike the AR-1 model. This difference in error dynamics results in high variance in the errors in AR-1 model.

The number of level crossings for a given interval is an important measure of likelihood of errors in a block size of data, such as a packet or a data frame. A model that captures this statistics is relevant in design of coding and interleaving schemes.

3.8 Summary

In most of the work in the literature, one time step dependent models such as, Markov chains have been used as a representation of wireless fading channels. In this chapter, the MC model based on the error threshold level is compared with AR models. By taking into account the physics of the process, it is shown that the Rayleigh channels require a minimum of two-step dependence and therefore an AR-2 model may be applicable. It is shown that while the MC model and the AR models capture the statistics averaged over a long period of time, at least second
order memory is required to capture the channel dynamics on a shorter time scale. Modeling accuracy of short-time scale characteristics is important when the packet level error behavior of the channel is considered. In the next chapter, the AR-2 model is augmented to use the channel sampling information for prediction purposes.

The next chapter examines AR models for channel prediction. Though AR-2 outperforms AR-1 in modeling second order features of the channel as shown in this chapter, the AR-1 model is also considered for comparison purposes. Markov chains are eliminated as a candidate model due to their deficiency in modeling second-order statistics.
CHAPTER 4

CHANNEL CONDITION FORECASTING

4.1 Introduction

A wireless transmitter and receiver are designed to exchange data in a standard transmission format. A number of standards are in use worldwide. For example, the IS-95 standard is used in USA primarily for voice communications. In Europe, the standard is GSM [2]. Among the many transmission protocol guidelines, the standards also specify the methods for end-systems to infer the channel behavior. Estimation of channel quality is typically done through the transmission of pilot or training symbols. The training sequence of given length may be transmitted in every transmission frame or interspersed between a fixed number of frames. The content, length and timing of the training sequence is known to the receiver. The training sequence allows the receiver to detect the channel and train the error cancellation and control schemes to adapt to the channel conditions. Since the process of training induces overhead on the channel resources, under-utilization of bandwidth results when the pilot symbols are transmitted at a frequency higher than the rate at which error-causing states occur in the channel. When the channel is sensed at a rate slower than the fading rate, the channel estimation errors increase. The choice of channel
estimation parameters is influenced by the temporal features of the channel.

For the Rayleigh fading channel, high correlation and oscillatory time dynamics influence the rate of occurrence of fading. For such a channel, the new information provided by successive samples is very small, particularly at high data-rates. Though the application of Markov chain models in channel prediction has not been investigated as a function of sampling intervals and/or forecast lead times exceeding one symbol interval, higher order AR models with time varying coefficients have been considered in [44] and [45]. It was found that to achieve long range prediction accuracy, channel sampling was required at twice the Doppler frequency irrespective of the data-rate of the application. In this chapter, a sampling scheme is designed to estimate the correlation structure of the channel on a long time-scale. The short term correlation is inferred through a local interpolation scheme. The choice of sampling rate is based upon the correlation metric of the channel and the transmitted SNR. A predictive model for flat fading Rayleigh channels is developed based on the proposed sampling scheme.

4.2 Channel Sampling

Consider a signal transmitted with symbol interval $\Delta t_s$. The channel is proposed to be sampled at times $t = L\Delta t_s$, $L = 1, 2, \cdots$. The selection of $L$, typically greater than 1 to isolate various channel states, is an important consideration in the model. For oscillatory signals with highest frequency $\omega_m = 2\pi f_m$ radians/s the sampling interval should be $0 < L\Delta t_s \leq 1/2\omega_m$ to satisfy the Nyquist theorem [35]. The effect of using the Nyquist rate $L = 1/(2\omega_m\Delta t_s)$ is examined for cellular and
wireless LAN applications.

Voice communications through cellular phones that use IS-95 standard operate at a data-rate of 9.6 kbps per connection using a carrier frequency of 1900 MHz. Wireless local area networks (WLAN) using the 802.11b standards operate at a carrier frequency of 2.4 GHz supporting a data-rate of 11 Mbps. Therefore, the symbol interval $\Delta t_s$ is typically 0.1 ms for cellular phones and is 0.1 $\mu$s in the WLAN environment. A cellular phone user moving at a vehicular speed of 25 miles/hr induces a Doppler frequency shift of 90 Hz. Consequently, $L = 1/(2\omega_m \Delta t_s) \approx 15$ bits. A wireless LAN deployed in an indoor environment may be subject to a Doppler frequency shift of 10 Hz when users move at a pedestrian speed of approximately 1.5 m/s. This requires the sampling interval of $L = 175\times10^3$ bits. In either case, this choice of $L$ samples the channel at time intervals where the channel correlation coefficient decays down to 0.8 and therefore channel estimation or prediction can lead to large errors. Therefore, sampling frequency smaller than the Nyquist rate must be considered to enable accurate prediction.

The choice of sampling lag is influenced by the correlation in the channel. Consider the correlation coefficient in the range 1.0 and 0.8. For correlation coefficients close to 1, the variance of the sampled values is very small. Consequently, the information content is proportionately small. Empirical observations show that at larger sampling lags where the correlation coefficient falls below 0.9, the variance of the samples is too high to enable channel prediction. The correlation coefficient values in the range [0.98 : 0.90] provide good channel estimates and eliminate information redundancy induced by heavily correlated samples. In the next section, the deter-
mination of $L$ is presented as a function of channel parameters: $f_m$, data rate and SNR.

4.3 Selection of Sampling Lag $L$

The objective is to find the largest value of sampling lag $L$ that allows prediction of channel fades with acceptable accuracy. The sampling lag $L$ is a critical model parameter. The sampling lag is a count of number of bits between the channel samples. In most applications the channel state is required rather than the exact attenuation value. The acceptable accuracy in such applications requires that the transitions across the threshold level that separates the fade and non-fade states be captured successfully. Therefore selection criteria for $L$ is focussed on channel state estimation, that is, transitions into a given amplitude regime.

The threshold $\chi_{TH}$ determined in Section 2.7.3 allowed to maintain a ratio of $P_e^{TH}/P_e = C\%$. This level classified the channel into fade and non-fade states. When applied to the Gaussian distribution of inphase (quadrature) component $y_n$, this corresponds to states

\[
s_1 \quad \text{if} \quad [-\infty \leq y_n < -\chi_{TH}]
\]
\[
s_2 \quad \text{if} \quad [-\chi_{TH} \leq y_n < \chi_{TH}]
\]
\[
s_3 \quad \text{if} \quad [\chi_{TH} \leq y_n < \infty]
\]

(4.1)

Thus, $y_n$ is partitioned into three output symbols $s_1$, $s_2$, $s_3$. Since, the fade states are entered only when the signal falls in the range $[-\chi_{TH}: \chi_{TH}]$, states $s_1$ and $s_3$ correspond to non-fade states. State $s_2$ is the fade state. The sampling lag should be selected to capture the events that transition the channel from fade to a non-fade
state and vice-versa. The choice of \( L \) may be based on the conditional expectation of \( y_n \). The conditional expectation based on the knowledge of one previous sample at lag \( L \) is

\[
\mu^{(1)} = E[y_n \mid y_{n-L}] = \int_{-\infty}^{\infty} y f_Y(y \mid y_{n-L}) \, dy
\]

and the conditional variance is

\[
\sigma^{2(1)} = Var(y_n \mid y_{n-L}) = \int_{-\infty}^{\infty} y^2 f_Y(y \mid y_{n-L}) \, dy - \left( E[y_n \mid y_{n-L}] \right)^2
\]

One may estimate \( \hat{y}_n \) based on first order memory to reside in the following range with a confidence of 75% [57]

\[
\hat{y}_n^{(1)} : [\mu^{(1)} \pm \sigma^{(1)}]
\]

The expectation of \( y_n \) conditioned on two previous samples at \( n - L \) and \( n - 2L \) is

\[
\mu^{(2)} = E[y_n \mid y_{n-L}, y_{n-2L}] = \int_{-\infty}^{\infty} y f_Y(y \mid y_{n-L}, y_{n-2L}) \, dy
\]

and the variance is

\[
\sigma^{2(2)} = Var(y_n \mid y_{n-L}, y_{n-2L}) = \int_{-\infty}^{\infty} y^2 f_Y(y \mid y_{n-L}, y_{n-2L}) \, dy - \left( E[y_n \mid y_{n-L}, y_{n-2L}] \right)^2
\]

The estimated value \( \hat{y}_{n(2)} \) based on the second order memory can be expected to fall in the following range with 75% confidence

\[
\hat{y}_{n(2)} : [\mu^{(2)} \pm \sigma^{(2)}]
\]

As \( L \) is increased, the error \( y_n - \hat{y}_n^{(1)} \) and \( y_n - \hat{y}_n^{(2)} \) will also increase. We conjecture that the error is acceptable as long as the state transitions are predictable.
The process transitions can occur as \( s_g \rightarrow s_b, \ s_g \rightarrow s_g, \ s_b \rightarrow s_g \) and \( s_b \rightarrow s_b \), where the fade state \( s_b = s_2 \) and the non-fade state \((s_1, s_2) : s_g\). The variation of \( \hat{y}_n^{(1)} \) and \( \hat{y}_n^{(2)} \) as a function of \( L \) is examined next.

4.3.1 Sampling Using First Order Memory

The lag \( L_g \) to model all the state transitions \( s_g \rightarrow s_b \) and \( s_g \rightarrow s_g \) that originate in \( s_g \), should satisfy

\[
L_g = \min [L_{g \rightarrow b}, L_{g \rightarrow g}] \tag{4.8}
\]

where

\[
L_{g \rightarrow b} = \max L : [\hat{y}_n^{(1)} \in s_b \ | \ y_{n-L} \in s_g]
\]
\[
L_{g \rightarrow g} = \max L : [\hat{y}_n^{(1)} \in s_g \ | \ y_{n-L} \in s_g]
\]

where \( L_{g \rightarrow b} \) represents the lag when \( y_{n-L} \in s_g, \ y_n \in s_b \) and \( L_{g \rightarrow g} \) is the lag when \( y_{n-L} \in s_g, \ y_n \in s_g \). Similar notations are used when transitions originate from \( s_b \).

The lag \( L_b \) for transitions originating from fade state \( s_b \) should be chosen such that

\[
L_b = \min [L_{b \rightarrow g}, L_{b \rightarrow b}] \tag{4.9}
\]

where

\[
L_{b \rightarrow g} = \max L : [\hat{y}_n^{(1)} \in s_g \ | \ y_{n-L} \in s_b]
\]
\[
L_{b \rightarrow b} = \max L : [\hat{y}_n^{(1)} \in s_b \ | \ y_{n-L} \in s_b]
\]

In order to capture all of the channel transitions, the maximum possible value of sampling \( L_{\text{max}} \) for the first order memory should be

\[
L_{\text{max}} = \min [L_g, L_b] \tag{4.10}
\]

The conditions in Eq. (4.8) and (4.9) are examined next.
In Fig. (4.1)(a)-(c) \( \hat{y}_n^{(1)} = \mu^{(1)} \pm \sigma^{(1)} \) is plotted as a function of \( L \). The values \( \mu^{(1)} \) and \( \sigma^{(1)} \) are obtained from Eq.(4.2) and Eq.(4.3) respectively using the numerical integration algorithm \( DECUHR \) [62]. The desired conditional PDF is given by Eq.(3.35). The state partitioning is done using Eq.(4.1). The state boundaries \( \pm \chi_{TH} = \pm 0.5 \) are marked in the figure by dark solid lines. The fade state corresponds to the region bounded by the solid lines and non-fade state extends on both sides of this region. The value selected for \( \chi_{TH} \) is the threshold resulting from \( \Omega_p = 1 \), Doppler frequency \( f_m = 90 \text{ Hz} \), symbol interval \( \Delta t_s = 72 \times 10^{-6} \) and transmitted SNR of 9 dB. The value \( \mu^{(1)} \) is marked by the dots and the vertical lines indicate \( \pm \sigma^{(1)} \).

When the \( y_{n-L} \in s_g \), the selection of \( L \) is specified by Eq.(4.8). The conditions are examined in (a) where \( y_{n-L} \in s_g, s_g = s_1 \) and also in (c) where \( y_{n-L} \in s_g, s_g = s_3 \).

For Rayleigh fading channels, once the process enters a state, it remains there for the duration governed by the inherent correlation characteristics. Since correlation reduces with increasing \( L \), the expectation \( \mu^{(1)} \) progresses from the non-fade into the fade state. However, due to high variance, \( \hat{y}_n^{(1)} = \mu^{(1)} \pm \sigma^{(1)} \), always fluctuates across
the state boundaries irrespective of the lag value as shown in Fig. 4.1(a) and (c). This implies that when only first order memory is used, no value of \( L \) can be chosen that would satisfy conditions specified by Eq.(4.8). Therefore, we conclude that the first order memory is insufficient to model the transitions leading from non-fade state to either fade or non-fade states.

Fig. 4.1(b) depicts the variation about \( \hat{y}_n^{(1)} \) when the channel is in a fade state at \( n-L \), that is \( y_{n-L} \in s_b \). This is the case specified by Eq.(4.9). The expectation \( \mu^{(1)} \) in this case is always zero. For \( L \leq 18 \), \( \hat{y}_n^{(1)} = \mu^{(1)} \pm \sigma^{(1)} \) lies within the fade state. This is because the variance \( \sigma^{(1)} \) is small enough so that the state boundaries are not crossed. We conclude that using first order memory, the likelihood of remaining in the fade state, once having entered it, may be correctly modeled.

Since no value of \( L \) can satisfy all the conditions in Eq.(4.10), the selection criteria needs to be modified to find \( L \) when only first order memory is allowed. The variance can be scaled by a constant factor \( A \), \( 0 < A < 1 \) replacing \( \sigma^{(1)} \) by \( A\sigma^{(1)} \). The scaling will result in a value of \( L \) by compromising the modeling accuracy in capturing state-transitions. Alternatively, we may continue to use \( \sigma^{(1)} \) as is, while recognizing the limitations in modeling the state transitions. The latter approach is used here. Henceforth, the criteria

\[
I_{\text{max}} = I_b
\]  

(4.11)

will be used when only the first order memory is available.
4.3.2 Sampling Using Second Order Memory

When the second order memory is used, the transitions are a function of states of $y_{n-L}$ and $y_{n-2L}$. Let $L_{gb}$ represent the lag when $y_{n-2L} \in s_g$ and $y_{n-L} \in s_b$ and $L_{gb-b}$ when $y_n \in s_b$. The notation for all the other combinations of the states assumed by $y_n$, $y_{n-L}$ and $y_{n-2L}$ are similarly obtained.

The lag $L_{gg}$ that models the transitions from non-fade state using the second order memory, that is when $y_{n-L}, y_{n-2L} \in s_g$ should satisfy

$$L_{gg} = \min[L_{gg-g}, L_{gg-b}]$$ \hspace{1cm} (4.12)

where

$L_{gg-g} = \max L : [\hat{y}_n^{(2)} \in s_g \mid y_{n-L}, y_{n-2L} \in s_g]$\hspace{1cm} (4.13)

$L_{gg-b} = \max L : [\hat{y}_n^{(2)} \in s_g \mid y_{n-L}, y_{n-2L} \in s_g]$\hspace{1cm} (4.14)

When the channel has already transitioned into non-fade state, that is $y_{n-L} \in s_g$ and $y_{n-2L} \in s_b$, $L$ should satisfy

$$L_{bg} = L_{bg-g}$$

where

$L_{bg-g} = \max L : [\hat{y}_n^{(2)} \in s_g \mid y_{n-L} \in s_g, y_{n-2L} \in s_b]$\hspace{1cm} (4.15)

assuming $P[\hat{y}_n^{(2)} \in s_b \mid y_{n-L} \in s_g, y_{n-2L} \in s_b] \approx 0$.

To be able to model the transitions originating in the fade state $s_b$, $L_{bb}$ should be

$$L_{bb} = \min[L_{bb-g}, L_{bb-b}]$$

where

$L_{bb-g} = \max L : [\hat{y}_n^{(2)} \in s_g \mid y_{n-L}, y_{n-2L} \in s_b]$\hspace{1cm} (4.16)

$L_{bb-b} = \max L : [\hat{y}_n^{(2)} \in s_b \mid y_{n-L}, y_{n-2L} \in s_b]$
When the channel has already transitioned into the fade state, that is \( y_{n-L} \in s_b \) and \( y_{n-2L} \in s_g \), \( L_{gb} \) should satisfy

\[
L_{gb} = L_{gb \rightarrow b} \tag{4.15}
\]

where \( L_{gb \rightarrow b} = \text{Max } L : \left[ \hat{y}_n^{(2)} \in s_b \mid y_{n-L} \in s_b, \ y_{n-2L} \in s_g \right] \)

The probability \( P \left[ \hat{y}_n^{(2)} \in s_g \mid y_{n-L} \in s_b, \ y_{n-2L} \in s_g \right] \approx 0 \) is assumed as the channel is highly correlated. In order to capture all the channel transitions, the maximum possible value of sampling parameter \( L_{max} \) for the second order memory should be

\[
L_{max} = \min [L_{gg}, L_{bb}, L_{gb}, L_{bg}] \tag{4.16}
\]

Next, the variation in \( \hat{y}_n^{(1)} \) and \( \hat{y}_n^{(2)} \) are examined as a function of \( L \).

The variation in \( \hat{y}_n^{(2)} = \mu^{(2)} + \sigma^{(2)} \) with respect to \( L_{gg} \) and \( L_{bg} \) is plotted in Fig. 4.2(a), (b) and with respect to \( L_{db} \) and \( L_{gb} \) in 4.3(a), (b). The values \( \mu^{(2)} \) and \( \sigma^{(2)} \) are obtained using numerical integration algorithm \textit{DECUHR} [62]. The desired second order conditional PDF is given by Eq.(3.39). The case when \( y_{n-L}, y_{n-2L} = s_3 \) is depicted in Fig. 4.2(a). The corresponding constraint on \( L \) is specified by Eq.(4.12).

Due to symmetry of the Gaussian PDF, the curve for \( y_{n-L}, y_{n-2L} \in s_1 \) is the mirror image of (a) reflected along the horizontal axis. In Fig. 4.2(b), the variation of \( L_{bg} \) is plotted. The constraint is given by Eq.(4.13). The variation in \( \hat{y}_n^{(2)} \) with \( L \) is shown when \( y_{n-L} \in s_g, \ y_{n-2L} \in s_b \) when \( s_g = s_1 \). The plot for the case when \( s_g = s_3 \) is a mirror image. Irrespective of the value of \( L \), \( \hat{y}_n^{(2)} \) resides outside the state boundary.

The variance \( \sigma^{2(2)} \) is very high in both cases resulting in state estimation errors and the criteria in Eq.(4.12) and Eq.(4.13) are not satisfied by any value of \( L \). Therefore forecast errors may result. For example, the violation of the constraints may result in
a forecast of fade state even when the actual process has transitioned into a non-fade state. This case is similar to Fig. 4.1(a), (c) where the first order memory was used. We conclude that even second order memory is insufficient to model the transition originating in the non-fade state.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure4.2a}
\includegraphics[width=0.5\textwidth]{figure4.2b}
\caption{\( \hat{y}^{(2)}_n \) vs \( L \) for \( \chi_{TH} = 0.5 \), \( SNR = 9dB \)}
\end{figure}

The case when process excursions originate in fade state \( s_b \) is examined next. The result for \( y_{n-L}, y_{n-2L} \in s_2 \) is plotted in Fig. 4.3(a). The value \( \mu^{(2)} \) is constant and equals zero in this case. The excursions of the process within \( \pm \sigma^{(2)} \) remain confined within the fade-state for \( L \leq 24 \). The variation of \( \hat{y}^{(2)}_n \) is similar to \( \hat{y}^{(1)}_n \) when the transitions start from state \( s_b \).

In Fig. 4.3(b) \( y_{n-L} \in s_b, y_{n-2L} \in s_g \) is plotted. The case when \( s_b = s_2 \) and \( s_g = s_3 \) is depicted. Due to symmetry, the curve when \( s_g = s_3 \) is the mirror image reflected along the horizontal axis. The examination of the figure shows that once the channel is already in fade state, the variance \( \sigma^{(2)} \) is very small and the value of \( \hat{y}^{(2)}_n \) resides in the fade state for \( 4 \leq L \leq 12 \). These values of \( L \) satisfy the constraints
Figure 4.3: $\hat{y}_n^{(2)}$ vs $L$ for $\chi_{TH} = 0.5$, $SNR = 9dB$

given by Eq.(4.14) and Eq.(4.15). This implies that when $4 \leq L \leq 12$ is chosen, the model can capture transitions from the fade to fade or non-fade state.

Since choice of $L$ does not satisfy the constraints for selection of $L_{gg}$, $L_{bg}$, the selection criteria for selecting $L_{max}$ is relaxed when the second order memory is available. The ideal requirement in Eq.(4.16) is modified to

$$L_{max} = \min[L_{bb}, L_{gb}]$$  \hspace{1cm} (4.17)

Henceforth, the value of $L_{max}$ is chosen using Eq.(4.17) when the second order memory is available.

4.3.3 Selected Values of Lag

The discussion thus far is specific only to $f_m = 90$ Hz, symbol interval $\Delta t_s = 72 \times 10^{-6}$, $SNR = 9$ dB and $\chi_{TH} = 0.5$. A broad range of parameter values is considered in this section. Fig. 4.4(a) shows $L$ as a function of transmitted SNR ranging 4 to 20dB. The Doppler frequency is fixed at $f_m = 90$ Hz. Fig. 4.4(b) shows $L$ for
Doppler frequency 10 to 180 Hz and $SNR = 9\ dB$. The symbol duration used for (a) and (b) is $\Delta t_s = 72\times10^{-6}$. The effect of varying the transmitted data-rate from 64 kbps to 6 Mbps is shown in (c) at $SNR = 9\ dB$ and $f_m = 90\ Hz$. The value of $L$ is selected using Eq. (4.11) when only the first order memory is available and Eq. (4.17) is used for the second order memory. Changing $SNR$ as in (a) results in changes in the threshold value $\chi_{TH}$ chosen for partitioning the PDF of the inphase component into states. The lag selection using first order memory is mostly insensitive to variations in the transmitted SNR. The value of $\chi_{TH}$ varies from 0.55 to 0.1 for SNR range 8 dB to 20 dB. Due to small variation in threshold range, the changes in the conditional expectation are not significant. The selection of $L$ using the second order memory exhibits increased sensitivity resulting in increased sampling rate for increasing SNR values. The amplitudes contributing to the given fraction $C\%$ of errors become progressively small as the SNR increases. Therefore higher resolution and consequently finer sampling is required as SNR is increased.

The influence of Doppler frequency is depicted in Fig. 4.4(b) on a log scale.
along horizontal and vertical axes. \( L \) decays as a power law with the Doppler frequency. The rate of decay in the Bessel correlation increases as Doppler frequency is increased. Consequently, the window of correlated data that may enable prediction progressively shrinks resulting in high sampling frequency requirements. Hence, the sampling lag \( L \) decreases with increase in the Doppler frequency. The selected lag value in each case corresponds to a correlation coefficient close to 0.95.

When the data rate is increased, the bit interval decreases. Consequently, the number of correlated bits increases. As shown in the Fig. 4.4(c), the increase in data-rates results in linearly increasing \( L \) when either first order or the second order memory is used for sampling.

The choice of suitable sampling scheme for Rayleigh fading channels as a function of the transmitted SNR, Doppler frequency and data-rate has not been investigated before to the best of our knowledge. The work by Hallen et al. [45] uses a sampling frequency equal to twice the Doppler frequency for AR model of order higher than 2. The influence of data-rate and SNR was not investigated in that work. The next section discusses how \( L \) is integrated into the AR model.

4.4 Autoregressive Models and Sampling

Once an appropriate value of \( L \) is selected, the AR process can be used to model every \( L^{th} \) sample and the samples \( nL : (n+1)L \) in between are linearly interpolated. As before, the inphase and quadrature components of the Rayleigh envelope are independently modeled. To illustrate the model setup, an arbitrary region of the inphase component is shown in the Fig. 4.5. The data-points \( y_k, k = 0, L, 2L, 3L, \cdots \) have a
correlation coefficient in the range \([0.98 : 0.90]\) and correspond to the points marked \(A, B, C\) in the figure. Intermediate \(L - 2\) data-points in the interval \([nL : (n + 1)L]\), end-points excluded, are heavily correlated with the correlation coefficient ranging \([1 : 0.98]\). The intermediate points in the range \([nL : (n + 1)L]\) are linearly interpolated using the interval boundary.

Recall that the second order autoregressive model AR-2 was found most appropriate for Rayleigh fading channels through the analysis of asymptotic expansion of ACF in Section 3.6. The first order AR-1 model is also considered here for comparison purposes.

The first order model is obtained by substituting the correlation coefficient \(\rho_1 = \rho_L\) into Eq.\((3.49)-(3.54)\) to yield

\[
y_n = \Phi_1 y_{n-L} + \eta_n
\]

\[
\Phi_1 = \rho_L \quad (4.19)
\]

\[
\sigma_N^2 = \frac{\Omega_p}{2} \left(1 - \rho_L^2\right)
\]

where \(\rho_L\) is given by Eq.\((3.34)\). The second order model is obtained by substituting
correlation coefficient $\rho_1 = \rho_L$ and $\rho_2 = \rho_{2L}$ in Eq.(3.57)-(3.61) to give

$$y_n = \Phi_1 y_{n-L} + \Phi_2 y_{n-2L} + \eta_n$$  \hspace{1cm} (4.20)

$$\Phi_1 = \frac{\rho_L (1 - \rho_{2L})}{1 - \rho_L^2}$$  \hspace{1cm} (4.21)

$$\Phi_2 = \frac{\rho_{2L} - \rho_L^2}{1 - \rho_L^2}$$

$$\sigma_N^2 = \frac{\Omega_p}{2} (1 - \Phi_1 \rho_L - \Phi_2 \rho_{2L})$$

While choice of $L$ allows flexibility in the model to capture correlations spanning different time-scales, the modeling accuracy decreases with increasing $L$. The model accuracy is a function of the noise variance $\sigma_N^2$. Fig. 4.6 depicts $\sigma_N^2$ of the first and second order AR processes as a function of the sampling lag $L$. The parameters are calculated using Eq.(4.19) for first order process and Eq.(4.21) for the second order process. The ACF $\rho_k$ is evaluated for a Doppler frequency of 90 Hz and a symbol interval $\Delta t_s = 72 \times 10^{-6}$. The variance of AR-1 as well as AR-2 processes increase

![Figure 4.6: $\sigma_N^2$ as a function of sampling lag](image)

with increase in sampling lag $L$. The process variance increases at a higher rate for AR-1 than for AR-2. Due to dependence only on one previous symbol, AR-1 loses
correlation information faster resulting in higher noise variance. AR-2 uses memory spanning two previous symbols, consequently providing better modeling accuracy.

When a very small value of \( L \) is chosen where the autocorrelation coefficient \( \rho_{yy} \approx 1 \), the variance \( \sigma_N^2 \) specified by Eq.(4.19) and (4.21) approaches zero. This may result in an unstable model and the stochastic process may not be represented correctly.

### 4.4.1 Linear Interpolation

The samples within the interval \([nL: (n + 1)L]\) are linearly interpolated using the interval boundary. The local samples are approximated by the following equation

\[
y_k = m_k + c, \quad nL < k < (n + 1)L
\]

The parameters slope \( m \) and the intercept \( c \) are evaluated using the end points \( y_{nL} \) and \( y_{(n+1)L} \) as follows

\[
m = \frac{y_{(n+1)L} - y_{nL}}{L} \quad (4.23)
\]

\[
c = y_{(n+1)L} - my_{nL}
\]

### 4.5 Forecasting

The selection scheme for choosing \( L \) specified in Section 4.3 is governed by the state-transitions among fade and non-fade states. However, when applied to AR processes, as explained in Section 4.4, continuous values for the inphase or the quadrature components are obtained. We conjecture that the AR process can be used
to estimate the future values assumed by the inphase and quadrature components by using the channel information collected \( L \) bits apart for a suitably chosen \( L \). The resulting Rayleigh envelope can be obtained by combining the inphase and the quadrature components. The application of an AR model for forecasting is discussed in this section.

Let \( n \) be the current time, \( \hat{y}_{n+F} \) be the forecast at lead time \( F \) and \( y_{n+F} \) be the actual value at time \( n + F \). Using Eq.(4.18) the forecast for AR-1 process is given by

\[
\hat{y}_{n+F} = \Phi_1 y_n, \quad F = kL, \ k = 1, 2, \ldots
\]

When the AR-2 model is used, Eq.(4.20) gives a forecast

\[
\hat{y}_{n+F} = \Phi_1 y_n + \Phi_2 y_{n-L}, \quad F = kL, \ k = 1, 2, \ldots
\]

The squared error for lead time \( F \) is

\[
e_n^2(F) = [y_{n+F} - \hat{y}_{n+F}]^2
\]

Using Eq.(3.42) the observation at time \( n + F \) also be obtained in terms of \( \psi_i \) as

\[
y_{n+F} = \eta_{n+F} + \sum_{j=1}^{n+F-1} \psi_j \eta_j
\]

Recall that \( \eta_j \) is a white noise process distributed as \( N(0, \sigma^2_N) \). Since \( E[\eta_j] = 0 \), the expectation \( E[y_{n+F}] = E[\hat{y}_{n+F}] = 0 \) and the error variance \( Var(e_n^2(F)) \) is equal to the the second moment of forecast error \( Var(e_n^2(F)) = E[e_n^2(F)] \). Substituting Eq.(4.27) in Eq.(4.26) and taking expectation on both sides gives the second moment of the forecast error. Since \( \eta_j \) is also uncorrelated, the expectation of all the cross
terms is zero. Therefore the error variance \( \text{Var}(e_n^2(F)) \) can be written as [61]

\[
\text{Var}(e_n^2(F)) = \left(1 + \sum_{j=1}^{F-1} \psi_j^2 \right) \sigma_N^2
\]  

(4.28)

To determine the forecast error variance for an AR process of given order, \( \psi_j \) should be obtained in terms of the AR coefficients \( \Phi_j \). Let \( B \) be the backward operator defined as

\[
By_n = y_{n-1}
\]  

(4.29)

\[
B^j y_n = y_{n-j}
\]

The AR-1 process can be expressed in terms of \( B \) as

\[
y_n \left(1 - B \Phi_1\right) = \eta_n
\]  

(4.30)

or

\[
y_n = \eta_n \left(1 - B \Phi_1\right)^{-1}
\]  

(4.31)

\[
= \sum_{j=0}^{\infty} \Phi_1^j \left(\eta_n B^j\right)
\]

Equating coefficients of \( \eta_j \) in Eq. (4.31) and Eq. (4.27) the value of \( \psi_j \) for the AR-1 process can be obtained as

\[
\psi_j = \Phi_1^j
\]  

(4.32)

The AR-2 process can be expressed in terms of \( B \) as

\[
y_n \left(1 - (B \Phi_1 + B^2 \Phi_2)\right) = \eta_n
\]  

(4.33)

or

\[
y_n = \eta_n \left(1 - (B \Phi_1 + B^2 \Phi_2)^{-1}\right)
\]  

(4.34)

\[
= \sum_{j=0}^{\infty} \left(B \Phi_1 + B^2 \Phi_2\right)^j \eta_n
\]
By substituting \( n = 0, 1 \ldots \) in Eq.(4.34) and Eq.(4.27) and comparing the coefficients, \( \psi_j \) for AR-2 process can be obtained as

\[
\psi_j = \psi_{j-1}\Phi_1 + \psi_{j-2}\Phi_2
\]

(4.35)

where \( \psi_i = 0, i < 1 \) and \( \psi_1 = \Phi_1 \). The forecast error variance \( \text{Var}(\epsilon_n^2(F)) \) can be calculated for AR-1 and AR-2 processes by substituting Eq.(4.32) and (4.35) respectively in Eq.(4.28). The variance \( \text{Var}(\epsilon_n^2(F)) \) can also be evaluated as a function of \( L \) by choosing \( \Phi_i \) as a function of \( L \) using Eq.(4.19) and (4.21). \( \text{Var}(\epsilon_n^2(F)) \) is plotted for lag \( L = 2, \ldots, 25 \) in Fig. 4.7(a) for AR-1 process and in (b) for the AR-2 process.

As one would expect, the forecast error variance increases with \( L \) for AR-1 as well as AR2. The rate of increase of \( \text{Var}(\epsilon_n^2(F)) \) for AR-1 process is higher compared to AR-2 process. Error variance for AR-1 and AR-2 models are compared for the same sampling lag \( L = 2 \) and \( L = 6 \) in Fig. 4.8(a) and (b) respectively. The forecast error variance for AR-1 process is considerably higher compared to AR-2 process for the same \( L \).
Since the prediction error varies with $L$, the criteria for selection of $L_{\text{max}}$ specified in Section 4.3 must be revised to account for prediction errors. The selection criteria specified by Eq.(4.11) for first order order AR model can be modified by estimating $\hat{y}^{(1)}_n$ as

$$\hat{y}^{(1)}_n = \mu^{(1)} + \sigma^{(1)}'$$

where $\sigma^{(1)}' = \sigma^{(1)} \pm \sqrt{\text{Var}(e^2_n(F))}$

and using the following estimate for criteria in Eq.(4.17) for second AR process

$$\hat{y}^{(2)}_n = \mu^{(2)} + \sigma^{(2)}'$$

where $\sigma^{(2)}' = \sigma^{(2)} \pm \sqrt{\text{Var}(e^2_n(F))}$

The model is applied for error control at the transmitter. Interleaving at the transmitter is considered for error control.

4.6 Block Errors with Interleaving

Rayleigh fading channels introduce correlated error bursts in the transmitted signal. An effective method to correct the burst errors is to use interleaving. An
interleaver is a device that rearranges the ordering of transmitted bits such that the burst errors appear random when the de-interleaver restores the original order. Interleaving delay $D$ is the separation between the successive bits in the interleaved sequence. To achieve an uncorrelated and reduced error process, channels with slowly decaying correlation require larger interleaving depth compared to the channels that exhibit a faster correlation decay rate. In order to decode the signal, the decoder is required to wait until all the successive bits in a block can be retrieved from the received sequence. Therefore, the decoding delay at the receiver increases as the interleaving depth is increased. The choice of $D$ is an important consideration in the design of an interleaving scheme. The probability of block errors in an interleaved sequence transmitted on the Rayleigh fading channel is investigated in this section. A block of size $n$ is declared in error if $m$ bits $0 < m \leq n$ fall in the error causing state. The states are defined by Eq. (4.1). The decision boundary for error $X_{TH}$ is influenced by the transmitted SNR as described in Section 2.7.3. The interleaving delay $D$ is chosen independently of the sampling lag $L$. While the values that $L$ can assume are restricted by requirements specified in Section 4.3, $D$ can assume values in the range $0 < D \leq 2/\omega_m$ to model the Bessel function correlation. The range of values that $D$ can assume come from the approximation of the Bessel correlation function by a cosine function form as explained in Section 3.6.

Let the data be interleaved prior to transmission with a delay $D$ and $n$ be the number of bits in a block. The de-interleaved sequence can be obtained by gathering $n$ bits in the received sequence that are $D$ bits apart. Alternatively, an AR process with parameters evaluated using $D^{th}$ order correlation can be used to generate a
de-interleaved sequence. Let $P(m, n)$ be the probability of $m$ errors in a block of $n$ bits. $P(m, n)$ is defined for inphase and quadrature components of the Rayleigh fading signal. To analyze $P(m, n)$, the probability distribution function of inphase-quadrature components of the received signal is divided into three states. The PDF is partitioned according to Eq.(4.1) using the threshold $\chi_{TH}$. Recall that the state $s_1$ and $s_3$ are non-error states and $s_2$ is the error state. Let $P_i(m, n)$ be the conditional probability of having $m$ errors in a block of size $n$ symbols under the condition the current state of the process is $s_i$

$$P_i(m, n) = Pr[m \text{ errors in } (1 : n) \text{ outcomes } | \ S_0 = s_i] \quad (4.38)$$

The total block error probability can be defined in terms of block error probabilities $P_i(m, n)$ as

$$P(m, n) = \sum_{i=1}^{3} P_i(m, n)\pi_i \quad (4.39)$$

where $\pi_i$ is the steady-state probability of being in state $s_i$. Evaluation of $P_i(m, n)$ can be done recursively by considering first order statistics for AR-1 and second order statistics for AR-2 process. The block error probability is derived using first and second order statistics in the following sections.

### 4.6.1 Block Errors for AR-1 Process

Let the conditional probability $P_{ij}$ be

$$P_{ij} = Pr(S_n = s_j \ | \ S_{n-1} = s_i) \quad (4.40)$$

and $S_n$ be the state assumed by the channel at instant $n$. The event 0 errors in $n = 1$ corresponds to $S_1 = s_g$ where $s_g = s_1$ or $s_2$. The event 1 error in $n = 1$ implies
that $S_1 = s_b$. The probability $P_i(m, n)$ can be decomposed as a summation of events involving error and non-error states considering outcome at instant $n = 1$ and and outcome in the interval $(2 : n)$

$$P_i(m, n) = P_i[S_1 = s_1] P_i[m \text{ errors in } 2 : n \text{ outcomes}]|S_1 = s_1]$$

$$+ P_i[S_1 = s_3] P_i[m \text{ errors in } 2 : n \text{ outcomes}]|S_1 = s_3]$$

$$+ P_i[S_1 = s_2] P_i[m - 1 \text{ errors in } 2 : n \text{ outcomes}]|S_1 = s_2]$$

Using the definition in Eq.(4.38) and substituting

$$P_i[m \text{ errors in } 2 : n \text{ outcomes}]|S_1 = s_1] = P_i(m, n - 1)$$

$$P_i[m \text{ errors in } 2 : n \text{ outcomes}]|S_1 = s_3] = P_3(m, n - 1)$$

$$P_i[m - 1 \text{ errors in } 2 : n \text{ outcomes}]|S_1 = s_2] = P_2(m - 1, n - 1)$$

Using Eq.(4.38) and (4.40) the following shorthand notation can be obtained

$$P_i[S_1 = s_1] = Pr[S_1 = s_1 | S_0 = s_i] = P_{i1}$$

$$P_i[S_1 = s_2] = Pr[S_1 = s_2 | S_0 = s_i] = P_{i2}$$

$$P_i[S_1 = s_3] = Pr[S_1 = s_3 | S_0 = s_i] = P_{i3}$$

Substituting Eq.(4.42) and (4.43) in Eq.(4.41) gives

$$P_i(m, n) = P_{i1} P_i(m, n - 1) + P_{i2} P_2(m - 1, n - 1) + P_{i3} P_3(m, n - 1)$$

Substituting Eq.(4.44) in Eq.(4.39) the total block error probability is

$$P(m, n) = \sum_{i=1}^{3} P_{i1} P_i(m, n - 1) + P_{i1} P_2(m - 1, n - 1) + P_{i1} P_3(m, n - 1)$$
The initial values for the recursion are the steady state probabilities

\[ P_j(0, 0) = \pi_j, \quad j = 1, 2, 3 \]  \hspace{1cm} (4.46)

The recursion is defined for \( n = 1\ldots\infty, \ m = 0\ldots n \) with the constraint

\[ P_i(m, n) = 0, \ m > n \]  \hspace{1cm} (4.47)

\[ P_i(m, n) = 0, \ m < 0 \text{ or } n < 0 \]

### 4.6.2 Block errors using AR-2 process

When the second order dependence is considered, there can be at the most 3 errors between \( n^{th} \) and \( (n-2)^{th} \) step. Let,

\[ P_{ijk} = P(S_n = s_k \mid S_{n-2} = s_i, \ S_{n-1} = s_j) \]  \hspace{1cm} (4.48)

The error process is influenced by the states assumed at instants \( n, \ n-1 \) and \( n-2 \). The cases resulting from various combination of outcomes at these instants are specified in Table 4.1. The block error probability in each case is obtained as follows.

**Case 1:** The outcomes are \( S_n, S_{n-1}, S_{n-2} = s_1 \text{ or } s_3 \). Therefore the block error probability is given by

\[ P_k(m, n) = P(m, n - 3) \sum_{i=1,3} \sum_{j=1,3} P_{ijk}, \quad k = 1, 3 \]  \hspace{1cm} (4.49)
<table>
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<th>State at n-1</th>
<th>State at n</th>
<th>Error count</th>
</tr>
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<td>$s_g$</td>
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<td>0 Errors</td>
</tr>
<tr>
<td>Case 2</td>
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<td>$s_g$</td>
<td>$s_b$</td>
<td>1 Error</td>
</tr>
<tr>
<td></td>
<td>$s_g$</td>
<td>$s_b$</td>
<td>$s_g$</td>
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</tr>
<tr>
<td></td>
<td>$s_b$</td>
<td>$s_g$</td>
<td>$s_g$</td>
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</tr>
<tr>
<td>Case 3</td>
<td>$s_g$</td>
<td>$s_b$</td>
<td>$s_b$</td>
<td>2 Errors</td>
</tr>
<tr>
<td></td>
<td>$s_b$</td>
<td>$s_b$</td>
<td>$s_g$</td>
<td></td>
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<tr>
<td></td>
<td>$s_b$</td>
<td>$s_g$</td>
<td>$s_b$</td>
<td></td>
</tr>
<tr>
<td>Case 4</td>
<td>$s_b$</td>
<td>$s_b$</td>
<td>$s_b$</td>
<td>3 Errors</td>
</tr>
</tbody>
</table>

Table 4.1: Pattern of states at \(n, n - 1, n - 2\) instants and error counts

**Case 2:** The outcomes are, \(S_{n-1}, S_{n-2} = s_1 \text{ or } s_3\) resulting in the block error probability given by

\[
P_k(m,n) = P(m - 1, n - 3) \sum_{i=1,3} \sum_{j=1,3} P_{ijk} \quad k = 2 \quad (4.50)
\]

\[
P_k(m,n) = P(m - 1, n - 3) \left[ \sum_{i=1,3} \sum_{j=2} P_{ijk} + \sum_{i=2} \sum_{j=1,3} P_{ijk} \right] \quad k = 1, 3
\]

**Case 3:** Two errors out of total \(m\) occur between \((n : n - 2)\) in this case giving

\[
P_k(m,n) = P(m - 2, n - 3) \sum_{i=2} \sum_{j=2} P_{ijk}, \quad k = 1, 3 \quad (4.51)
\]

\[
P_k(m,n) = P(m - 1, n - 3) \left[ \sum_{i=1,3} \sum_{j=2} P_{ijk} + \sum_{i=2} \sum_{j=1,3} P_{ijk} \right] \quad k = 2
\]

**Case 4:** All of the \((n - 2)_{\text{th}}\) to \(n\) outcomes are in error

\[
P_k(m,n) = P(m - 3, n - 3) P_{ijk}, \quad i = j = k = 2 \quad (4.52)
\]

All the cases mentioned above are mutually exclusive, therefore the total probability of block errors can be obtained by adding the error probability in each case. The recursion is defined for \(n = 2...\infty, m = 0,...,n\) with constraints in Eq.(4.47).
The initial values for the recursion are defined using Eq.(4.46)-(4.42). The initial conditions for \( m = 0, n = 0 \) are

\[
P_j(0, 0) = \pi_j \quad j = 1, 2, 3 \tag{4.53}
\]

for \( m = 0, n = 1 \)

\[
P_j(0, 1) = \sum_{i=1}^{3} P_i(0, 0) P_{ij} \quad j = 1, 3 \tag{4.54}
\]

\[
P_j(0, 1) = 0 \quad j = 2
\]

and for \( m = 1, n = 1 \)

\[
P_j(1, 1) = 0 \quad j = 1, 3 \tag{4.55}
\]

\[
P_j(1, 1) = \sum_{i=1}^{3} P_i(0, 0) P_{ij}, \quad j = 2
\]

### 4.7 Results

Recall that the Eq.(4.11) and (4.17) were set-up to determine state transitions and not the actual values assumed by the process. However, due to continuous values given by AR processes, the model can also be used to forecast the amplitudes as well. In this section, the relative accuracy of the AR models in predicting \( \hat{y}_{n+F} \), \( F = L, 2L, \cdots \) is evaluated. The effect of linearly interpolating the intermediate samples in the interval \([\hat{y}_{n+F} : y_n]\) are also considered. Since autocorrelation function is an important feature of the model in capturing process-dynamics, the distortion in the correlation features of the forecasts are also examined. The application of AR models in determining block error probabilities are discussed.
4.7.1 Forecasting Using AR Models

Performance of AR-1 and AR-2 models in predicting the inphase component is evaluated in this section. One step forecasts are considered for a sampling lag $L = 12$. The sampling lag corresponds to the Doppler frequency $f_m = 90$ Hz, symbol duration of $\Delta t_s = 72 \times 10^{-6}$ and a threshold of $\chi_{TH} = 0.5$. The $\chi_{TH}$ corresponds to the signal at $SNR = 9$ dB. The same $L$ is selected for AR-1 and AR-2 processes. The forecasts are compared using the inphase component and the Rayleigh distributed trace as reference. The reference trace is generated using the Jake’s simulator described in Appendix-B.

4.7.2 1-Step Forecast

![Inphase AR1 Linear Interpolation](image1)

![Inphase AR2 Linear Interpolation](image2)

(a) AR-1  
(b) AR-2

Figure 4.9: Estimated inphase component with reference trace, $L = 12$, prediction for $F = L$

One step forecast $F = L$ along with the trace for an arbitrarily chosen section of the data is shown in Fig. 4.9(a) for AR-1 model and in (b) for the AR-2 model.
The forecast is done in steps of $L$ using Eq.(4.24) for AR-1 model and Eq.(4.25) for the AR-2 model. The intermediate points in the interval $[nL : n(L + F)]$ are linearly interpolated using Eq.(4.22) and (4.23). In the figure, the dark clusters show the forecast using AR models and the interpolated points. The starting point of the clusters is the observation and the other end is the forecast. The points in between are linearly interpolated. Due to slowly decaying correlation in the AR-1 model, the forecasts are very close to the observations. The interpolated points therefore form a dense cluster. The interpolated values using AR-2 model follow the trend of the data. The plots show that the estimates match fairly well with the reference trace for AR-2 process where monotonic increase or decrease in the amplitude occurs. Forecasts using AR-1 model exhibit much larger deviations from the trace. The result of combining inphase and quadrature components to obtain the Rayleigh distributed envelope is shown in Fig. 4.10. The inphase and quadrature components are generated using the AR model and the result is combined using Eq.(3.16) to obtain the estimate for the Rayleigh distributed envelope. The AR-2 model outperforms the AR-1 model. Some deviations in Fig. 4.12(b) and Fig. 4.10(b) are observed where the level stays almost constant with time or a change in trend occurs. The effect of long-range forecast is analyzed next.

4.7.3 Long Range Forecast Using AR-2 Model

The long-range estimates can be obtained either by using coarse samples and performing 1-step forecast or through a multi-step forecast using fine samples as shown in Fig. 4.11. The effect of multiple step forecast is examined in Fig. 4.12.
The forecast $\hat{y}_{n+F}$ for a fixed value of $n + F$ is obtained by suitably selecting $F$ and $L$. For multi-step forecast, $L = 12$, $F = 2L$ is chosen and for 1-step forecast, $L = 24$, $F = L$ is chosen. In both the cases, the estimated value at $n + F = 24$ is obtained. The examination of the figure shows that the two step forecast exhibits larger error whenever the amplitude is maintained almost constant. Deterioration in performance can be attributed to inaccuracies in the autocorrelation function of the forecasts when 2 step prediction is obtained. As a result of sinusoidal correlation inherent in the Rayleigh fading channels, a 1-step forecast using coarse samples performs better than the multi-step forecast obtained using fine samples.

4.7.4 Autocorrelation of Forecasts

The data is generated using the AR-2 model for $L = 12, F = L, 2L$ step forecast and autocorrelation of the forecasts generated is evaluated. Fig. 4.13(a) compares the ACF for the AR-2 model with the Bessel function when $F = L$. One
Figure 4.11: Long-range forecast

step prediction preserves correlation information. Fig. 4.13(b) shows the changes in ACF when \( F = 2L \). Though the trend of ACF is captured well, deviations in autocorrelation function are evident. Larger forecast steps induce more distortions in the correlation function. Therefore higher forecast errors can be expected with increasing \( F \).

4.7.5 Estimation of Block Errors

The accuracy of block-error probability distribution function \( P(m, n) \) determined using the recursion formula described in section 4.6 is evaluated. \( P(m, n) \) is the probability of having \( m \) bits in errors in a block size of \( n \). In Fig. 4.14, \( P(m, n) \) evaluated using simulated traces and recursion formula is plotted for Doppler frequency \( f_m = 90 \) Hz for symbol interval \( \Delta t_s = 72 \times 10^{-6} \), interleaving delay \( D = 2 \) and block size \( n = 50 \). The decision boundary for the error is the threshold value described in Section 2.7.3. The states corresponding to error and non-error condition are described by Eq.(4.1). The threshold value of \( \chi_{TH} = 0.1 \) is used which corresponds to a \( SNR = 20 \) dB. The simulated trace of inphase component is generated using the Jake’s simulator described in Appendix-B. AR-1 and AR-2 traces are
generated using the parameters described by Eq.(4.19) and Eq.(4.21) respectively.

The probability $P(m, n)$ of number of errors $m$ in a block size $n$ is determined. In
Fig. 4.14(a) $P(m, n)$ is shown as obtained by simulation at an interleaving depth
$D = 2$. An error is assumed if the inphase component resides in state $s_2$. The AR-2
model follows $P(m, n)$ for inphase component very closely, whereas AR-1 model de-
viates considerably. Using the AR-1 model $P(m, n)$ is under-estimated for $m \leq 10$
and over-estimated for $m > 10$. Fig. 4.14(b) shows $P(m, n)$ evaluated using the
recursion formula. AR-1 and AR-2 both deviate from the simulated result. The
autocorrelation coefficient for $D = 2$ is close to 0.99. Since the recursion formula
includes memory only of first order for AR-1 and second order for AR-2, enough
memory is not included for small $D$. In Fig. 4.15(a),(b) plots of the $P(m, n)$ evalu-
ated using simulation and recursion for $D = 30$ are shown. At $D = 30$, the process
autocorrelation is 0.66. The inclusion of short memory in recursion is sufficient to
calculate $P(m, n)$ accurately when the correlation coefficient is $\leq 0.66$. Therefore,
when correlation coefficient is large, one has to include memory of the order \( i \) such that the correlation coefficient value \( \rho_{yy}(\Delta t) \leq 0.65 \) is modeled.

### 4.7.6 Effect of Interleaving

The interleaving depth required on fading channels is influenced by the rate of decay in ACF. The ACF on Rayleigh fading channels is influenced by the Doppler frequency and the application data-rate. Often, the applications are designed to achieve a certain error probability. The application driven error metrics impose requirements on the correction capability required for a given block-size. In this section, the selection of block-size, interleaving depth and correction capability requirements are investigated for Rayleigh fading channels. The probability \( P(m,n) \) of having \( m \) errors in a block-size of \( n \) bits for fixed interleaving delays \( D \) can be evaluated using the recursion formula described in Section 4.6 when the correlation coefficient is about 0.6. Simulation can be used when the correlation coefficient is in
Figure 4.14: $P(m,n)$ for block-size $n = 50$, interleaving depth $D = 2$, $\chi_{TH} = 0.1$

the range $[1 : 0.7]$.

In Fig. 4.16(a) $P(m, 50)$ for Doppler frequency $f_m = 90$ is plotted for interleaving delay $D = 10, 50, 30$ using the simulated AR-2 trace. The figure also shows $P(m, n)$ for $D = 100$ evaluated using the simulated trace for inphase component generated using the Jake’s simulator. The threshold value $\chi_{TH} = 0.1$ is used which corresponds to a $SNR = 20$ dB. The function $P(m, n)$ assumes a stable shape for $D \geq 30$. The Doppler frequency of $f_m = 10$ hz is considered in Fig. 4.16(b). Much larger values of $D \geq 300$ are required for $P(m, n)$ to assume steady shape. In general, a larger interleaving depth is required when the fading is characterized by a slowly decaying correlation.

The change in SNR results in change in the error threshold $\chi_{TH}$ for the block-error process. Therefore changing SNR influences the shape of PDF $P(m, n)$. $P(m, n)$ for $f_m = 90$ Hz and $\Delta t_s = 72x10^{-6}$ is depicted in Fig. 4.17 for SNR of 20 dB in (a) and for 9 dB in (b). The threshold of 0.1 is observed when $SNR = 20$ dB
and threshold is 0.5 for $SNR = 9$ dB. A block size $n = 50$ is used for both cases. At 9 dB, the PDF $P(m, n)$ ceases to change for $D \geq 50$ as against $D \geq 30$ for 20 dB, suggesting longer interleaving depth requirement at small SNR values. This is due to high error rate at low SNR values.

The correction capability required for a given block-size to achieve a block error probability $P(m, n) \leq 1 \times 10^{-5}$ is examined next. The influence of Doppler shift and transmitted SNR are considered. At $f_m = 90$ Hz as shown in Fig. 4.16 (a), the errors $m$ are less than 15 for the specified error probability when an interleaving delay $D \geq 30$ is chosen. For $D = 10$, the specified error probability is reached for $m > 35$. If an error correcting capability of 15 bits is provided when interleaving delay is $D \geq 30$ and a capability of 35 bits when $D = 10$, a $P(m, n) \leq 10^{-5}$ can be expected. As shown in Fig. 4.16 (b), the required correction capability is 15 bits for $D \geq 300$ and 30 bits for $D = 100$ when Doppler frequency is 10 Hz. When $D$ is increased from 30 to 50 in (a) and 300 to 1000 in (b), the block error probability
distribution remains unchanged. This behavior suggests that for a fixed block size, an optimum interleaving depth is required to obtain un-correlated errors. Increasing the interleaving depth beyond that value does not improve block-error performance. As shown in Fig. 4.17(a), (b) the correction capability of 20 bits is required when $SNR = 20$ dB as against 35 bit when $SNR = 9$ dB.

In general, the choice of number of correction bits required is governed not only by the block-size and interleaving delay but also by the channel parameters: Doppler frequency, symbol interval and SNR. The choice for optimum interleaving delay is determined for $f_m = 10, 90$ Hz next for a fixed SNR and symbol interval. The effect of block-sizes is also considered.

In Fig. 4.18, the maximum number of bits in error are plotted against interleaving delays for block sizes of 50, 100 and 200 bits. In Fig. 4.18(a) the Doppler frequency of 10 Hz and in (b) the Doppler frequency of 90 Hz is used. The symbol interval in both cases is $\Delta t_s = 72x10^{-6}$ and $SNR = 9$ dB. In Fig. 4.18(a), the bits
in error are almost constant for interleaving delay $D > 250$. The bit error count is about 18 for block-size $n = 50$, 25 for $n = 100$ and about 40 for $n = 200$. Hence, a delay $D \geq 250$ is required for Doppler frequency $f_m = 10$ Hz at fixed symbol interval, irrespective of the block sizes chosen. If the correction capability is provided for the aforementioned error count, a block size of $n = 50$ requires $18/50 = 36\%$, $n = 100$ requires 25% and $n = 200$ requires 20% bits to be corrected. By increasing the block size by a factor of two from $n = 50$ to 100 the requirements for the correction capability improved by 9% and an improvement of 5% was obtained when the block size was changed again by the same factor from $n = 100$ to 200. Fig. 4.19 shows percentage of correction capability for $P(m,n) \leq 10^{-5}$ against block size on a log scale along horizontal and vertical axis. The fraction of correction capability requirement is influenced by the block size and decreases inversely as a power law. Thus, the fraction of bits that require correction decrease with increasing block-size. Therefore large block-size may be favored. The interleaving delay must be chosen as
Figure 4.18: Error-bits in block vs interleaving delay, $\Delta t_s = 72 \times 10^{-6}$, $\chi_{TH} = 0.1$

a function of Doppler frequency for a fixed symbol duration. To select the number of correction bits, the number of bits in error for a given block size may be used as a criteria.

In this section, it is shown that $D$ must be chosen based on Doppler frequency for fixed data-rate. To achieve a given block error probability, large block-sizes are favored. This is because the number of bits required to be corrected for given probability of error decreases with block sizes. Very small interleaving delays cannot be analyzed for highly correlated channels using the model. This is the case when the autocorrelation coefficient approaches unity rendering $\sigma_X^2$ in Eq.(4.18) and (4.20) close to zero.

4.8 Summary

In this chapter, a sampling scheme is presented to model channel transitions originating in the fade state. The sampling lag is evaluated as a function of the
Figure 4.19: % correction required for $P(m, n) \leq 10^{-5}$ vs block size $n$

channel parameters. The sampling interval is integrated with the AR model for channel predictions. It is shown that slow as well as fast fading channels can be reasonably predicted using suitable sampling. It is demonstrated that a one step forecast using coarse samples is advantageous over multi-step forecast obtained using fine samples. A recursive algorithm is presented to compute block-errors. The model is applied for error control at the transmitter. The error control through interleaving is considered. Using the model, interleaving parameters are obtained as a function of channel features.

In the next chapter, the modeling of multipath channels is considered. The model is applied for equalization through prediction.
CHAPTER 5

PREDICTIVE MODEL FOR MULTIPATH FAISING CHANNELS

5.1 Introduction

A linear multipath channel is represented using the channel impulse response (CIR). The CIR allows the response of the channel to an arbitrary input signal to be evaluated. For a wireless channel, the impulse response consists of a signal received through the line of sight (LOS) path followed by the echoes of the transmitted signal. The line of sight path has the highest energy and enables clear signal detection. The reverberations typically constitute a profile of decaying energy values. These are the non line-of-sight (NLOS) arrivals. The multipath components contribute as intersymbol interference. When the channel is sampled at a given rate, the CIR can be represented as a summation of delayed impulses of varying amplitudes. In the absence of the transmitter/receiver mobility or channel variations, the CIR magnitude and phase of each path may be assumed fixed for a T-R separation. In reality, wireless channels almost always exhibit time-variation. Relative motion of transmitter-receiver pair or movement within the propagation medium causes the CIR to change with time. For slow-fading channels, the impulse response amplitudes change slowly. Depending on the data-rate of the system, the channel may appear
to remain almost stationary over several symbol durations on slow-fading channels. Fast fading channels on the other hand are manifest as rapid temporal variations in the CIR. In this chapter, the autoregressive model developed is applied to modeling path variations of the CIR. The Kalman filter is applied to predict channel variations and this information is used to equalize the effects of ISI.

5.2 Channel Impulse Response

The transmitted signal \( s(t) \) and received signal \( r(t) \) are sampled at discrete time intervals spaced \( \Delta t \) apart. The samples \( \zeta_n \) and \( r_n \) are

\[
\zeta_n = s(n\Delta t), \quad r_n = r(n\Delta t) \quad (5.1)
\]

When the channel is modeled as a linear system the impulse response \( h_k, k = 0, 1, \ldots \) is a unique characteristic of the channel. A causal system generates a response

\[
r_k = \sum_{i=0}^{\infty} h_i \, \zeta_{k-i} \quad (5.2)
\]

In terms of input and output \( z \)-transforms,

\[
H(z) = \frac{R(z)}{S(z)} \quad (5.3)
\]

\( H(z) \) can be expressed as a ratio between two polynomials

\[
H(z) = \frac{\sum_{i=0}^{N} \alpha_i z^{-i}}{1 - \sum_{i=1}^{N_p} \beta_i z^{-i}} \quad (5.4)
\]

where \( \alpha_i \) and \( \beta_i \) are characteristic constants of the channel. If the CIR is of finite length with \( M + 1 \) samples and the effect of Gaussian noise is included, the received signal is

\[
r_k = \sum_{i=0}^{M} h_i \, \zeta_{k-i} + \eta_k \quad (5.5)
\]
In practice, the CIR is temporally varying. That is to say, the amplitude and phase of \( i^{th} \) tap \( h_i \) varies with time. The rate of variation depends upon the Doppler frequency shift influencing the channel. When \( h_i^k \) is the \( i^{th} \) tap at time \( k \), Eq.(5.5) can be extended for time varying channels as follows

\[
 r_k = \sum_{i=0}^{M} h_i^k \, \zeta_{k-i} + \eta_k \tag{5.6}
\]

### 5.3 Impulse Response of Fading Channels

Using Eq.(2.57) and (2.24), the received signal for single input and single output Rayleigh fading channels can be written as

\[
r(t) = \left[ y(t) + jz(t) \right] s(t) + \eta(t) \tag{5.7}
\]

where

\[
y(t) = \sum_n a_n \ s(t - \tau_n) \ \cos(2\pi \phi_n(t))
\]

\[
z(t) = \sum_n a_n \ s(t - \tau_n) \ \sin(2\pi \phi_n(t))
\]

It is shown in Section 2.3 that \( y(t) \) and \( z(t) \) are Gaussian distributed and the envelope \( x(t) = \sqrt{y^2(t) + z^2(t)} \) is Rayleigh distributed. In the discrete domain the signal can be written as

\[
r_k = \left[ y_k + jz_k \right] \zeta_k + \eta_k \tag{5.8}
\]

For multipath channels with \( M \) paths

\[
r_k = \sum_{i=0}^{M} \zeta_{k-i} \ h_i^k + \eta_k \tag{5.9}
\]
Here each tap, \( h_i^k \) is be assumed to be Rayleigh distributed with average power determined by the mean \( \overline{t}_i \). The AR models proposed in the previous chapters are applicable to inphase and quadrature components of the CIR. Therefore it is useful to express the CIR in the following form

\[
h_i^k = g_i^k + j f_i^k
\]  
(5.10)

Since \( h_i^k \) is Rayleigh distributed, \( g_i^k \) and \( f_i^k \) are Gaussian with equal variance. Expressing the transmitted signal also in quadrature form

\[
\zeta_i = p_i + j q_i
\]  
(5.11)

Substituting Eq.(5.10) and (5.11) in Eq.(5.6) the complex received signal can be expressed as

\[
r_k = \sum_{i=0}^{M} \left( g_i^k + j f_i^k \right) \left( p_{k-i} + j q_{k-i} \right) + \eta_k
\]  
(5.12)

Let the signal be transmitted with energy \( e_b \) using BPSK modulation scheme. The transmitted signal is \( p_i = \sqrt{e_b} \), \( q_i = 0 \) when the pulse is positive and \( p_i = -\sqrt{e_b} \), \( q_i = 0 \) when the pulse is negative. For BPSK modulation, the complex received signal can be simplified further. Let the received signal \( r_k \) and noise \( \eta_k \) be represented as

\[
r_k = u_k + j v_k
\]  
(5.13)

\[
\eta_k = \mu_k + j \nu_k
\]  
(5.14)

For BPSK, the real and imaginary parts of \( r_k \) can be written as

\[
v_k = \sum_{i=0}^{N} g_{k-i}^k p_i + \mu_k
\]  
(5.15)

\[
u_k = \sum_{i=0}^{N} f_{k-i}^k p_i + \nu_k
\]
5.4 Characterization of Time Variant CIR

In this work the mean value of the time variant CIR is assumed to be given by the pole-zero model in Eq.(5.4). Let \( \bar{h}_i \) be the mean value of the \( i^{th} \) channel tap. The mean \( \bar{h}_i \) can be obtained from the given pole-zero model. Let the inphase CIR component \( g_i \) be distributed as \( N(0, \sigma_{G_i}^2) \) and the quadrature component be \( f_i : N(0, \sigma_{F_i}^2) \). The variance \( \sigma_{G_i}^2 \) and \( \sigma_{F_i}^2 \) can be obtained using Eq.(2.46) by setting \( \sigma_{F_i}^2 = \sigma_{G_i}^2 = \frac{\sigma_i^2}{2} \) and \( x_{mean} = \bar{h}_i \). For illustration purposes, consider a one pole model

\[
H(z) = \frac{1}{1 - \beta z^{-1}}
\]  
(5.16)

The mean values of the impulse response taps \( \bar{h}_k \) can be written as

\[
\bar{h}_0 = \delta(0)
\]
(5.17)

\[
\bar{h}_i = \beta \bar{h}_{i-1}
\]

Therefore, the variance of inphase/quadrature components of the \( i^{th} \) CIR tap using Eq.(2.46) is

\[
\sigma_{G_i}^2 = \sigma_{F_i}^2 = \bar{h}_i^2 \frac{2}{\pi}
\]
(5.18)

All the channel taps are assumed to be subject to the same Doppler frequency. The AR parameters for the CIR taps are obtained using the normalized correlation coefficients using Eq.(4.19) for AR-1 and Eq.(4.21) for AR-2 model. Since \( \sigma_{G_i}^2 \) differs for each tap, the AR model for each tap differs in the noise variance of the AR process.
5.5 State Space Formulation

The communication system is modeled as a multiple input single output system. Inphase and quadrature components are dealt with separately. At an instant $k$, the vector $L_k$ is the input, $G_k$ is the inphase component of CIR and vector $F_k$ contains the quadrature component of CIR. All the vectors are in column format. The dimension and the value of vector elements are influenced by the order of AR model used. Formulation for AR-1 and AR-2 models are discussed separately in the following section. Eq.(5.15) can be written in matrix form as

$$u_k = L_k' G_k + \mu_k$$  \hspace{1cm} (5.19)

$$v_k = L_k' F_k + \eta_k$$

where ‘ indicates matrix transpose,

The dimension of the state space representation depends upon the order of pole-zero (PZ) model and the number of taps used in the CIR. The order of PZ representation is influenced by the CIR bandwidth. The smaller the bandwidth, the fewer are the poles and zeros required to represent the CIR accurately. In general, the length of CIR is infinite, but the number of taps with significant amplitude depend upon the decay rate of CIR. The finite length CIR is obtained by truncating the infinite length CIR to retain the significant taps. The faster the CIR decays, fewer are the taps required to retain the significant values. The CIR is assumed to be known for the given environment.
5.5.1 First Order Model

Using the first order model defined by Eq.(4.18) the \(i^{th}\) tap at time \(k\) can be written in terms of the AR coefficient as

\[
g_i^k = \Phi_1 g_i^{k-L} + \epsilon_i^k
\]  

(5.20)

where \(\Phi_1\) denotes the autoregressive coefficient calculated at a sampling lag \(L\) and \(\epsilon_j(k)\) denotes the autoregressive model error for \(i^{th}\) tap at time \(k\). The AR coefficient \(\Phi_1\) is the same for all the taps under the assumption that the taps exhibit same correlation behavior. Since AR noise is influenced by the variance of the modeled process, only the noise changes between taps. Recall that taps have different variance given by Eq.(5.18). Therefore the model for each of the \(M\) CIR taps can be written as

\[
g_0^k = \Phi_1 g_0^{k-L} + \epsilon_0^k
\]  

(5.21)

\[
g_1^k = \Phi_1 g_1^{k-L} + \epsilon_1^k
\]  

\[
\vdots
\]

\[
g_M^k = \Phi_1 g_M^{k-L} + \epsilon_M^k
\]

Let all the \(M\) CIR taps at instant \(k\) be aggregated in a \(M\times1\) column vector \(G_k\) where

\[
G_k = [g_0^k \ g_1^k \ \cdots \ g_M^k]^T
\]  

(5.22)

Using \(G_k\), Eq.(5.21) can be written as

\[
G_k = T \ G_{k-L} + E_k
\]  

(5.23)
where $E_k$ is $M \times 1$ vector containing the error terms of the AR model given by

$$
E_k = \begin{bmatrix}
\varepsilon_0^k & \varepsilon_1^k & \cdots & \varepsilon_M^k
\end{bmatrix}'
$$

The transition matrix $T$ is also an $M \times 1$ matrix given by

$$
T = \begin{bmatrix}
\Phi_1 & 0 & 0 & \cdots & 0 \\
0 & \Phi_1 & 0 & \cdots & 0 \\
0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & \cdots & \Phi_M
\end{bmatrix}
$$

To satisfy the Eq. (5.19), $M \times 1$ input vector is

$$
L_k = \begin{bmatrix}
p_k & p_{k-1} & \cdots & p_{k-M}
\end{bmatrix}'
$$

### 5.5.2 Second Order Model

Using the second order model defined by Eq. (4.20) the state transition equation for the $i^{th}$ tap at time $k$ can be written as

$$
g_i^k = \Phi_1 g_i^{k-L} + \Phi_2 g_i^{k-2L} + \varepsilon_i^k
$$

Eq. (5.27) can be obtained in matrix form as

$$
\begin{bmatrix}
g_i^k \\
g_i^{k-L}
\end{bmatrix} = \begin{bmatrix}
\Phi_1 & \Phi_2 \\
1 & 0
\end{bmatrix} \begin{bmatrix}
g_i^{k-L} \\
g_i^{k-2L}
\end{bmatrix} + \begin{bmatrix}
\varepsilon_i^k \\
0
\end{bmatrix}
$$

Hence there are 2 states corresponding to each channel tap. The CIR can be aggregated in a $2M \times 1$ column vector

$$
G_k = \begin{bmatrix}
g_0^k & g_0^{k-L} \\
g_1^k & g_1^{k-L} \\
g_2^k & g_2^{k-L} \\
\vdots & \vdots \\
g_M^k & g_M^{k-L}
\end{bmatrix}'
$$
and the AR model error can be aggregated in $2M \times 1$ vector

$$E_k = \begin{bmatrix}
\varepsilon_0^k & 0 & \varepsilon_1^k & 0 & \varepsilon_2^k & 0 & \cdots & \varepsilon_M^k & 0 \\
tap 0 & tap 1 & tap 2 & \cdots & tap M
\end{bmatrix}^T \quad (5.30)$$

The contribution of each tap is shown in the above equations. Using Eq.(5.29) and (5.30) the state transition equation can be written as

$$G_k = T \ G_{k-L} + E_k \quad (5.31)$$

The transition matrix $T$ is a $2M \times 2M$ matrix given by

$$T = \begin{bmatrix}
T_0 & 0 & 0 & \cdots & 0 \\
0 & T_1 & 0 & \cdots & 0 \\
0 & 0 & 0 & \ddots & 0 \\
0 & 0 & 0 & \cdots & T_M
\end{bmatrix} \quad (5.32)$$

and $T_1 = T_j = \begin{bmatrix} \phi_1 & \phi_2 \\ 1 & 0 \end{bmatrix}$

To satisfy Eq.(5.19), the input is a $2M \times 1$ vector given by

$$I_k = \begin{bmatrix} p_k & 0 & p_{k-1} & 0 & \cdots & p_{k-M} & 0 \end{bmatrix}^T \quad (5.33)$$

5.6 Kalman Filter

The state-space formulation given in Eq.(5.19) can be used to extend the AR model to multipath channels. The Kalman filter formulation is used to estimate the inphase component $\mathbf{G}_k$ and the quadrature component $\mathbf{E}_k$ of the CIR. The Kalman
filter presentation in this section closely follows [63]. The presentation is specific for inphase component only. Formulation can be easily extended to quadrature component by replacing $G_k$ by $E_k$. The measurement equation for $G_k$ is the multiple input, single output Eq.(5.19)

$$u_k = I_k^T G_k + \mu_k$$

(5.34)

The prediction equation is based on the AR models and is given by

$$G_k = T G_{k-L} + E_k$$

(5.35)

Values in $T$ are chosen based on the model parameters. Let the covariance matrix of the noise $E_k$ be $Q$. Since the noise is an uncorrelated Gaussian process, only the diagonal terms are non-zero. For the first order model,

$$Q_{ii} = \sigma^2_{G_i}$$

(5.36)

where, $\sigma^2_{G_i}$ is the model variance for $i^{th}$ tap. Under the assumption that the row/column indexing starts from 1, for the second order model

$$Q_{ii} = \begin{cases} 
\sigma^2_{G_i} & \text{if } i \text{ is odd} \\
0 & \text{if } i \text{ is even}
\end{cases}$$

(5.37)

Value of $\sigma^2_{G_i}$ is given by Eq.(5.18).

Let $\hat{G}_{k-1}$ denote the optimal estimator of $G_{k-1}$ at time $k-1$ based on observations upto and including $u_{k-1}$. Let $R_k$ be the covariance matrix of estimation error. $R_k$ can be obtained as

$$R_{k-1} = E \left[ (G_{k-1} - \hat{G}_{k-1}) (G_{k-1} - \hat{G}_{k-1})^T \right]$$

(5.38)
Given $\hat{G}_{k-1}$ and $R_{k-1}$, the optimal estimator of $G_k$ is given by

$$\hat{G}_{k|k-1} = T \hat{G}_{k-1}$$  \hfill (5.39)$$

when the prediction is carried out for all $k = 1, 2, ..., K$. The covariance matrix of the estimation error is

$$R_{k|k-1} = T R_{k-1} T' + Q$$  \hfill (5.40)$$

Eq.(5.39) and (5.40) comprise the prediction equations. The update equations are

$$\hat{G}_k = \hat{G}_{k|k-1} + R_{k|k-1} L_k A_k^{-1} (u_k - L_k' \hat{G}_{k|k-1})$$  \hfill (5.41)$$

and

$$R_k = R_{k|k-1} - R_{k|k-1} L_k A_k^{-1} L_k' R_{k|k-1}$$  \hfill (5.42)$$

where $A_k$ is a scalar obtained as

$$A_k = L_k' R_{k|k-1} L_k + \sigma^2_M$$  \hfill (5.43)$$

and $\sigma^2_M$ is the variance of the inphase component of noise $\mu_k$ which is given by Eq.(5.14).

5.7 Observability and Controllability

Conditions for observability of the system in transition are [63]

$$\text{rank}[L, T'L, \ldots, (T')^{M-1} L] = M$$  \hfill (5.44)$$

For the AR-1 process, $L$ is $M \times 1$ and $T$ is a $M \times M$ matrix and for AR-2 process $L'$ is a $2M \times 1$ and $T$ is a $2M \times 2M$ matrix. The rank for the AR-2 process should be $2M$.

Conditions for controllability are [63]

$$\text{rank}[Q, TQ, \ldots, (T)^{M-1}Q] = M$$  \hfill (5.45)$$
where \( Q \) and \( T \) are \( M \times M \) matrices for the first order model and are \( 2M \times 2M \) for the second order model. It can be shown that both the first and the second order models are observable. First order model is completely controllable and the second order model has only two controllable states.

### 5.8 Inter-Symbol Interference

Inter-symbol interference depends upon the rate of decay of the CIR in comparison to symbol duration. Let \( T \) be the reverberation time. Reverberation time is time taken for signal strength to decay to the noise floor, that is, to \(-60 \) dB relative to the transmitted signal energy. If the symbol duration \( \Delta t \geq T \), the ISI is negligible. When \( \Delta t < T \), the desired signal gets corrupted by the reflections of previous transmissions. Inter-symbol interference occurs when the received symbol is no longer distinguishable at the receiver from delayed arrivals. The strength of interfering components is influenced by the rate of CIR decay. The interference noise decreases with increasing decay in CIR. Out of \( n \) multipath arrivals let \( k^{th} \) symbol be the desired received symbol. The arrivals from \( 0 \) to \( k - 1 \) represent interference due to past transmissions. Arrivals corresponding to \( k + 1 \) to \( \infty \) represents interfering echoes of the desired symbol. Using Eq.(5.5) the received signal can be split into a sum of two terms

\[
r_k = \zeta_{k-i} h_i + \sum_{j=-\infty, j \neq i}^{\infty} \zeta_{k-j} h_j + \eta(k)
\]  

(5.46)

The first term represents the desired symbol received at the \( i^{th} \) instant and the second term represents the collective interference by the neighbouring symbols. To mitigate effects of ISI, one approach towards signal detection is to use Rake receivers.
Rake receivers detect the received symbol as a joint outcome of several neighbouring symbols. Another approach is to use equalizers. The equalization of time-varying channels using decision feedback equalizer (DFE) and equalization using predictive models is presented in this section.

5.9 Equalization

Equalizers are designed as inverse filters of CIR. The equalizer coefficients are chosen such that the desired symbol $\zeta_i$ is amplified for detection and signal echoes $\zeta_j$ for $j = 0 \ldots \infty, j \neq i$ are canceled. The likelihood of incorrect decision increases when the instantaneous strength of the desired symbol is rendered very small during channel fade. The equalizers are designed for very slow fading channels. The equalizer coefficients need to be periodically adjusted to adapt to channel variations. Effectiveness of equalizer under time varying conditions is a function of rate of convergence of equalizer coefficients to an optimum (or near optimum) value relative to rate of variations in the channel. Equalization using predictive models is based upon estimated channel coefficients. The only requirement is the channel sampling for prediction purposes. The technique can also be used for fast fading channels. The trade-off is higher channel sampling rate required compared to slowly fading channels.

5.9.1 Decision Feedback Equalizer

Let the terms in the Eq.(5.46) be reordered so that the desired symbol becomes the $6^{th}$ symbol. Using the new ordering, the summation can be split into non-causal
and causal terms as follows

\[ r_k = \sum_{j=-\infty}^{-1} \zeta_{k-j} h_j + \zeta_k h_0 + \sum_{j=1}^{\infty} \zeta_{k-j} h_j \quad (5.47) \]

When the detection is synchronized to \( h_0 \), the non-causal part \( j = -\infty, \cdots, -2, -1 \) corresponds to the interference due to the symbols transmitted after the desired symbol. The causal section \( j = 1, 2, \cdots, \infty \) corresponds to the interference due to already detected symbols that were transmitted prior to the desired symbol. In order to cancel the ISI, the decision feedback equalizer contains two transversal filters: feedforward section and feedback section. The feedforward section cancels the interference from non-causal symbols and the feedback filter cancels the interference from already detected symbols. Input to the feedforward section is the received signal sequence \( r_k \) and the input to the feedback section is the detected symbol sequence \( \hat{s}(k) \). For \( K_1 \) feedforward taps and \( K_2 \) feedback taps the equalizer output is given by

\[ \hat{c}_k = \sum_{j=-K_1}^{0} c_j r_{k-j} + \sum_{j=1}^{K_2} b_j \hat{\zeta}_{k-j} \quad (5.48) \]

where \( c_j \) is the feedforward and \( b_j \) is the feedback filter coefficient. The optimum values for tap coefficients can be obtained by choosing the coefficients to make the individual contributions from causal part \( j = \infty, \cdots, -2, -1 \) and non-causal part \( j = 1, 2, \cdots, \infty \) zero \([2]\). The feed forward filter coefficients are obtained by minimizing mean square error between input and detected symbol. The minimization gives equalizer feedforward tap coefficients in terms of cross-correlation between input symbol and channel output and auto-correlation between the channel output. The is the well known Weiner solution \([35]\) and gives feedforward filter coefficients as a
set of linear equations

\[ \sum_{j=-K_1}^{0} \xi_{lj} c_j = h_{-l}, \quad l = -K_1, \cdots, 1, 0 \]  \hspace{1cm} (5.49)

where,

\[ \xi_{lj} = \sum_{m=0}^{-l} h_m h_{m+l-j} + N_0 \delta_{lj}, \quad l, j = -k_1, \cdots, -1, 0 \]  \hspace{1cm} (5.50)

The feedback filter coefficients are given in terms of feedforward filter coefficients

\[ b_k = \sum_{j=-K_1}^{0} c_j h_{k-j}, \quad k = 1, 2, \ldots, K_2 \]  \hspace{1cm} (5.51)

The above solutions for filter coefficients require knowledge of CIR. The CIR is not usually known. However, the correlation between the output symbols can be recursively evaluated by using the channel output. The least-mean-square (LMS) algorithm [35] can be employed to evaluate the equalizer coefficients recursively.

5.9.2 Equalization Using Predictive Model

The predictive model gives the estimates of the CIR. Let \( \hat{h}_j^k \) be the estimate for \( j^{th} \) CIR tap at time \( k \). With known CIR tap values, the estimate of the transmitted symbol \( \hat{s}(k) \) corresponding to \( i^{th} \) tap can be obtained by canceling the ISI using

\[ \hat{\xi}_k = \frac{r_k - \sum_{j=-\infty}^{\infty} \hat{h}_j \hat{\xi}_{k-j}}{\hat{h}_i} \]  \hspace{1cm} (5.52)

\( \hat{\xi}_{k-j} \) corresponds to the detected symbol for previous transmissions.

5.10 Model Performance

The model performance is evaluated using simulation. The mean CIR is a one pole model evaluated using \( N_z = 0, N_p = 1, \beta_1 = 0.5 \) and number of channel taps are
$M = 1, \cdots, 4$. The variance of inphase and quadrature components are $\sigma_{G_i}^2$ and $\sigma_{F_i}^2$ given by Eq.(5.18). The channel operates at a symbol interval of $72 \times 10^{-6}$ seconds with Doppler frequency of 10 Hz. Different values of sampling lag $L$ are selected. The AR parameters for selected $L$ is given by Eq.(4.21). For a given SNR=$\epsilon_b/N_0$, the signal energy $\epsilon_b = 1$ for all simulation runs.

5.10.1 Prediction Error

Prediction performance using the Kalman filter is evaluated. The model is updated at the data-points every $L$ bits apart. At the current time $k$, the $i^{th}$ tap $g^k_i$ is the correct value and sample $g^{k+L}_i$ is predicted. The points $g^n_i$ for $n = k + 1, \cdots, k + L - 1$ are linearly interpolated. The mean squared error between the actual and predicted symbols is plotted for each channel tap in Fig. 5.1. Linearly interpolated data-points are excluded from mean error evaluation. Fig. 5.1(a) depicts the variation in the mean squared error for SNR=$20 \ dB$ for slow fading channel with
\(f_m = 10\) Hz and (b) plots the MSE for fast fading channels with \(f_m = 90\) Hz and 
\(\Delta t_s = 72 \times 10^{-6}\) using AR-1 and AR-2 models. The value \(L = 105\) for \(f_m = 10\) and \(L = 12\) for \(f_m = 90\). These values are consistent with the selection criteria in

Eq.(4.11) for AR-1 and Eq.(4.17) for AR2. Mean squared estimation error and one standard deviation about the mean is plotted for CIR taps. The mean value for all the taps is zero. The variance of MSE is very high for tap 1 and progressively decreases. This is because the variance \(\sigma^2_{G_i}\) for \(i = 1, \cdots, 4\) progressively decreases, thereby reducing the variance of the AR model and hence the MSE. The variance \(\sigma^2_{G_i}\) for AR-1 model is larger compared to AR2, hence higher mean square error for AR-1 results.

The influence of decreasing \(L\) is shown in Fig. 5.2(a) and (b). The lag \(L = 52\) for \(f_m = 10\) Hz and \(L = 6\) for \(f_m = 90\) Hz. Here, \(L\) is half of the those used in Fig. 5.1. For fast fading channels, halving the rate reduces the variance of MSE considerably, but for slow fading channel, the reduction is smaller. Due to slow change in correlation behavior, further reduction in sampling lag is required at low Doppler frequencies to reduce MSE variance.

It can be concluded from the results in this section that the sampling lag \(L\) selected to model the state transitions is not adequate to predict the exact amplitude values. Smaller lag value, in general close to 0.99 should be chosen to enable prediction of exact amplitudes.
Figure 5.2: Mean square error (MSE) variation with $L$, 1 pole CIR $\beta = 0.5$, $\Delta t_s = 72 \times 10^{-6}$

5.10.2 Equalization

The predicted values are used in order to cancel out the ISI due to multipath. ISI cancellation is done using Eq.(5.52). The detection is carried out on the symbols after cancellation of ISI and the average error rate is plotted against transmitted SNR in Fig. 5.3. The predictive model is run using the AR-2 model with parameters corresponding to the lags $L = 5, 10, 20$. Training bits of length $M$, where $M$ is the CIR length, are sent at every sampling lag. The average bit error performance is compared with the decision feedback equalizer. The equalizer is implemented using the LMS algorithm. To accommodate the time-varying channel, the equalizer is trained constantly. The channel is fixed initially and equalizer coefficients are initialized with a very small relaxation parameter value of 0.005. The relaxation parameter value is increased to 0.09 once the channel variations are initiated after coefficient initialization. The bit errors using predictive model and the DFE are
Figure 5.3: Average BER after equalization using predictive model and decision feedback equalizer

compared with the case when the channel is completely known. Equalization for completely known channel is also done using Eq. (5.52) by replacing the estimated values \( \hat{h} \) with the actual values \( h \). The desired symbol corresponds to the 0\(^{th} \) tap in all cases. It is clear that the prediction based equalization performs better than the constantly trained DFE for all the lag values. The bit-errors diverge for \( \text{SNR} \geq 30 \text{ dB} \) for \( L = 20 \). For high lag values, the modeling error exceeds the noise error variance. The resolution due to linear interpolation provides insufficient accuracy resulting in larger prediction errors. Therefore ISI cancellation is ineffective. Decreasing the lag further or increasing the model order from second to higher order can achieve performance improvements.

The bit errors are still very high for the predictive model and also for the case when the CIR is completely known. During channel variations, there are instances when the channel goes in deep fades. This occurs when the tap corresponding to the
desired symbol has an amplitude close to the noise floor. Bit error performance can be improved when transmission back-off is implemented during deep-fades.

The equalization using the predictive model when the known training symbols are used is compared with the case when detected symbols are used at every sampling lag. The result is shown in Fig. 5.4. For small lag values the difference in performance is negligible, implying high prediction accuracy for small lags.

5.11 Summary

In this chapter, the AR model is extended to time varying multipath channels. The taps of the finite length CIR are assumed to be mutually independent and time varying. Each tap is assumed to be Rayleigh distributed with equal Doppler shift. Thus, individual taps are identically correlated. The mean power profile of the CIR is obtained by a pole-zero model. The state-space representation of the
CIR is obtained. The inphase and quadrature components of the CIR represent the states. The state-space has dimension $2M$ for AR-1 and $4M$ for AR-2 when $M$ CIR taps are considered. The state transitions are modeled using the AR model. Due to the assumption of identical correlation behavior, the regression coefficients of the AR process are the same for all taps. The noise variance of the AR process is proportional to the mean power in the tap. The CIR is predicted using a Kalman filter. It is shown that in order to predict the exact value of the CIR taps, the channel needs to be sampled at lags $L$ that are much finer than those required for forecasting channel state transitions. For a suitable choice of sampling lag, a fast fading channel can also be reasonably predicted. A one step prediction of CIR taps in increments of $L$ and linear interpolation in the interval $[nL : (n + 1)L]$ is performed. The predicted values are used for the equalization of a time varying slow fading channel. The error statistics are compared with a continuously trained DFE. The equalization performance is analyzed when the channel is sampled by using $M$ training symbols at lag $L$ and also when detected values are used instead. It is shown that equalization performance improves by $> 25\%$ when the predictive model is used.
CHAPTER 6

CONCLUSIONS

6.1 Conclusions

The time varying behavior is a characteristic feature of wireless channels. The signal modulation, channel coding, error control and signal detection schemes should be adapted to varying channel conditions to achieve significant performance improvements. The prediction of channel behavior plays a decisive role in the implementation of such adaptive error control schemes. In this thesis, the problem of predicting the behavior of Rayleigh channels is considered for flat and frequency selective cases. Prediction performance under slow and fast fading conditions is examined. In this thesis, Markov chain and AR channel models are analyzed for channel characterization.

The Rayleigh fading channel is mapped into error and non-error states by computing an error threshold $\chi_{TH}$. The threshold $\chi_{TH}$ is the value of the Rayleigh envelope that captures $C\%$ of total errors. The fraction $0 < C < 1$ can be chosen depending upon an average bit error performance metric $P_e$ and $\chi_{TH}$ as a function of SNR.

The threshold $\chi_{TH}$ is applied to map the Rayleigh PDF into 2-state MC. The
first and second order AR processes are applied to model the inphase and quadrature components of the Rayleigh distributed envelope. The parameters for the AR-1 and AR-2 models are derived using conditional statistics of the multivariate Gaussian distribution function. The AR parameters are shown to be a function of channel parameters: data-rate, Doppler frequency and transmitted SNR. The relative merit of MC and AR models is compared with respect to time averaged error statistics. Since the errors that occur within a block of data are an important consideration in designing coding and interleaving schemes, the block error statistics are also investigated. While average statistics such as average bit error probability and level crossing rate are modeled well by all the aforementioned models, the block errors are modeled accurately only by the AR-2 process. This is because Markov chains and AR-1 under and overestimate the channel ACF. The same performance can be expected even if multiple-states in the MC are considered. This is because, irrespective of the number of states, the MC uses a first order memory and therefore cannot model the correlation behavior. On the other hand, the additional degree of freedom in AR-2 allows the characteristic Bessel function ACF of the Rayleigh fading channels to be modeled well for the lags before the first zero of the Bessel function occurs. Traditionally, first order Markov chain models have been used in characterizing stochastic features of the channel. It is shown in this work that Markov chain models are inadequate for modeling the behavior of the channel over a small time-scale. To capture variations of the channel over a short time duration, second and higher order models are necessary.

The application of AR models for channel prediction is discussed. The predic-
tion is based on the error threshold $\chi_{TH}$. A sampling scheme is developed to model the channel state transitions from fade to non-fade states and vice-versa. In order to improve bandwidth utilization, a threshold based sampling scheme is proposed. The goal of the proposed scheme is to avoid sampling the channel under heavily correlated conditions as incremental information provided by correlated samples is very small. A moderate value of lag $L$ is chosen such that the channel state-transitions can be modeled. The samples between the lag values are linearly interpolated. The sampling scheme can be adapted to variations in channel and transmission parameters. Prediction accuracy is shown to vary inversely with increase in sampling intervals.

The sampling information is integrated with the AR model. The AR-2 process is found to be an adequate model when the channel is sub-sampled to obtain the samples with the correlation coefficient close to 0.95. The prediction using the model is compared with the simulated trace. The results are presented for fast fading channels with a Doppler frequency of 90 Hz corresponding to a mobile speed of 25 miles/hr. The model is found satisfactory when the inphase/quadrature components exhibit monotonic increasing or decreasing trend. Prediction errors are higher when the level of inphase/quadrature exhibit trend reversals. The same model behavior is observed when the receiver is moving at a pedestrian speed of 1.5 meters/sec with Doppler frequency of 10 Hz except that the larger values of lags can be chosen at small Doppler frequency.

The model is applied to evaluate block-error statistics and the effect of interleaving on Rayleigh fading channels. Using simulation, the interleaving depth is determined for fixed channel parameters and/or error performance metric.
The AR model is also extended to time-varying multipath environments. A pole-zero model transfer function is assumed to represent the fading channel. The channel amplitudes are time-variant and follow Rayleigh statistics. The tap variations are independent of each other. The variance of each tap is obtained by jointly considering the pole-zero model and the Rayleigh statistics. The multipath channels are characterized by a multiple-input-single-output system. A state-space representation of multipath channel is presented such that, the CIR tap amplitudes inphase and quadrature components represent the states and the autoregressive model governs the state transitions. The state-space representation is used in the Kalman filter for prediction of the CIR in time. The model performance in multipath environments is evaluated when one step forecast is applied for channel equalization. The increase in sampling lag $L$ is shown to increase the mean square prediction error. The model is inadequate when the transmitted SNR exceeds 30 dB. At high SNR, the prediction error is higher than the channel error resulting in poor channel equalization. The performance is compared with a decision feedback equalizer that trains continuously. The average bit error probability is considerably lower when the predictive model is used.

6.2 Limitations

The degrees of freedom allowed in AR-2 cannot model state transitions originating from the non-fade state. Once the system has already transitioned into the fade state, the model performs well in forecasting the channel behavior.

Since the model is set-up to estimate the inphase and quadrature components
separately, the update equation in Kalman filter requires estimation of envelope as well as phase. The complexity in implementation increases due to separate estimation of inphase and quadrature components.

The space complexity of the state-space representation is in general governed by the number of channel taps used. Since the inphase and quadrature components are separately modeled, the space complexity increases by a factor of 2 for AR-1 and by a factor of 4 for AR-2 models. The time complexity of the prediction algorithm is a function of the underlying Kalman filter. The Kalman filter relies heavily on matrix manipulation resulting in an exponential increase in number of operations required per output with increasing number of channel taps. Using techniques such as singular-value-decomposition, time complexity of Kalman filter can be improved.

6.3 Contributions

Most of the work in the literature has considered one time step dependent Markov chain models to sample the channel at very close intervals to predict the error states. These models are inefficient and not physically motivated. In this work it is shown that Rayleigh fading channels require at-least a minimum of two-step dependence to capture even the most basic channel dynamics. By taking into account the physics in the process it is shown that a suitable sampling lag can be chosen to predict the channel transitions from the fade state.

Almost always in the literature, Markov parameters are independent of the channel and system parameters such as Doppler frequency and SNR. While the channels may be designed for fixed data-rate and SNR values, the Doppler frequency
usually changes over time requiring time varying parameters to characterize the channel distortions. AR models proposed in this work use the received data-sequence to estimate time-varying regression coefficients. This approach requires long training periods for channel estimation. The training based approach is adequate only for channels exhibiting high correlation. Such channels are characterized by small Doppler frequencies. For fast fading channels, the training based approach may provide outdated estimates when the channel varies at a rate faster than the training frequency. The model proposed in this thesis uses parameters defined as a function of channel and transmission parameters to ensure efficient bandwidth utilization. The channel sampling scheme is scalable with channel and transmission parameters. This work demonstrates that the model can also be applied to estimate the fast fading channel variations that result when the receiver moves at vehicular speeds.

The AR model is applied to estimate the channel states. It is shown that the coarsely spaced channel samples applied to one-step forecast is superior to finely spaced channel samples applied to multi-step forecast. The model is proved to be useful in selection of interleaving depth and block-sizes as a function of channel parameters. Equalizer performance improvement of $> 25\%$ is obtained using the model.

6.4 Recommendations for Future Work

It is shown that prediction errors in the second order model exceeds channel noise when the signal is transmitted with high SNR. As a result, equalization using the predictive model has higher detection errors for high SNR. The second order
model operating with the smallest lag value of 1 also is insufficient to achieve desired accuracy in predictions. To address the deficiency, a higher order model can be used at the cost of increased complexity. The optimum order of the required model needs further investigation. The effect of inaccurate channel estimates or the Doppler shift on prediction accuracy needs further investigation. More research is required to test the model for non-coherent detection. Improvement in adaptive schemes such as error control, rate-control, power control etc. by using the proposed approach needs evaluation.

In order to operate correctly, the model requires accurate information regarding inphase and quadrature components. This requirement may be the biggest bottleneck of the model for applications in the systems where the information regarding inphase/quadrature components may not be extractable from the channel output. An approach to extract inphase and quadrature components from observed channel output needs to be developed for such systems.

Issues with implementation of the predictive model in WLAN/Cellular devices requires further investigation. In the model developed in this work, Doppler frequency and the multipath delay are assumed constant. In practical implementations, the assumption is rarely valid. The effect of variations in Doppler spectrum and multipath delay must be included should the model be implemented for practical systems.

Channel variations are predominantly stochastic but also exhibit some degree of determinism. In case of Rayleigh fading channels, the deterministic behavior is manifest in the Doppler frequency shift. The Doppler frequency induces sinusoidal
correlation in the channel variations and allows predictions. The future work may involve characterization of specific radio propagation environments in terms of their deterministic components. A class of environment specific models may be developed to represent wireless channels.
APPENDIX A

PDF OF SIGNAL RECEIVED ON RAYLEIGH FADING CHANNEL

The received signal on an AWGN channel is given by Eq.(2.57) and

\[ r(t) = x(t)e^{j\varphi(t)} + \eta(t) \quad (A.1) \]

From Eq.(2.45) the Rayleigh PDF is given by

\[ f_X(x) = \begin{cases} \frac{2x}{\sigma_p^2} e^{-\frac{x^2}{2\sigma_p^2}} & x \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (A.2) \]

The Gaussian PDF \( N(0, \frac{N_0}{2}) \) is given by Eq.(2.55)

\[ f_N(\eta) = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{\eta^2}{4N_0}} \quad (A.3) \]

Assuming independence of noise and fading processes, the PDF of the received signal can be evaluated as the convolution of Rayleigh and Gaussian PDF. Hence

\[ f_R(r|\varphi) = f_X(x) * f_N(\eta) \quad (A.4) \]

The convolution can be evaluated by Fourier transforming the Gaussian and Rayleigh PDF, evaluating the product of the transforms and taking the inverse transform of the result. Using \( \xi \) as the Fourier domain variable, the received PDF \( F_R(j\xi) \) is

\[ F_R(j\xi) = F_X(j\xi) \cdot F_N(j\xi) \quad (A.5) \]
where $F_N(j\xi)$ is the Fourier transforms of Gaussian PDF given by

$$F_N(j\xi) = \int_{-\infty}^{\infty} f_N(\eta) \; e^{-j\xi\eta} \; d\eta \quad (A.6)$$

and $F_X(j\xi)$ is the Fourier transforms of Rayleigh PDF given by

$$F_X(j\xi) = \int_{-\infty}^{\infty} f_X(x) \; e^{-j\xi x} \; dx \quad (A.7)$$

The PDF of the received signal can be obtained by taking inverse Fourier transform as follows

$$f_R(r|\varphi) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F_R(j\xi) \; e^{i\xi r} \; d\xi \quad (A.8)$$

### A.1 Fourier Transform of Gaussian PDF

Substituting Eq.(A.3) in Eq.(A.6), the Fourier transform of Gaussian PDF can be written as

$$F_N(j\xi) = \frac{1}{\sqrt{\pi} N_0} \int_{-\infty}^{\infty} e^{\frac{-p^2}{\sqrt{N_0}}} e^{-j\xi\eta} \; d\eta \quad (A.9)$$

The integral can be evaluated using the following identity [59] (p.355)

$$\int_{-\infty}^{\infty} e^{-p^2x^2+q^2} \; dx = e^{\frac{q^2}{|p|}} \sqrt{\frac{\pi}{|p|}} \quad (A.10)$$

Comparing Eq.(A.9) and (A.10) the values are

$$p^2 = \frac{1}{N_0}; \quad |p| = \frac{1}{\sqrt{N_0}} \quad (A.11)$$

$$q = j\xi; \quad q^2 = -\xi^2$$
Substituting the above values in right-hand-side of Eq.(A.10) gives the result of the integral in Eq.(A.9). The result is

\[ F_N(j\xi) = e^{-\frac{\xi^2}{2}} \]  (A.12)

### A.2 Rayleigh Transform of Rayleigh PDF

Substituting Eq.(A.2) in Eq.(A.7), the Fourier transform of Rayleigh distribution can be written as

\[ F_X(j\xi) = \frac{2}{\Omega_p} \int_0^\infty x \, e^{-\frac{\xi^2}{2\Omega_p}} \, j\xi x \, dx \]  (A.13)

Notice that the lower limit of integration is changed from \(-\infty\) to 0, as Rayleigh PDF is zero for \(x \leq 0\). The above equation can be solved using the following result [59] (p.382)

\[ \int_0^\infty x^{\nu-1} \, e^{-\beta x^2-\gamma x} \, dx = (2\beta)^{-\nu/2} \Gamma(\nu) \exp\left(\frac{\gamma^2}{8\beta}\right) \, D_{-\nu}\left(\frac{\gamma}{\sqrt{2\beta}}\right) , \quad (A.14) \]

\[ \Re[\beta] > 0, \quad \Re[\nu] > 0 \]

The function \(D_{\nu}(\psi)\) is the parabolic-cylinder function. Comparing Eq.(A.13) and (A.14) the following values are obtained

\[ \nu = 2, \quad \Gamma(\nu) = \Gamma(2) = 1, \quad \beta = \frac{1}{\Omega_p}, \quad \gamma = j\xi \]

\[ \frac{\gamma^2}{8\beta} = \frac{-\xi^2\Omega_p}{8}, \quad (2\beta)^{-\nu/2} = \frac{\Omega_p}{2}, \quad \frac{\gamma}{\sqrt{2\beta}} = j\xi \sqrt{\frac{\Omega_p}{2}} \]

\(F_X(j\xi)\) can be evaluated as

\[ F_X(j\xi) = \exp \left[ -\frac{\xi^2 \Omega_p}{8} \right] \, D_{-2}\left(j\xi \sqrt{\frac{\Omega_p}{2}}\right) \]  (A.15)
A.3 Received Signal PDF

Substituting Eq.(A.12) and (A.15) in Eq.(A.5)

\[ F_R(j\xi) = \exp\left(\frac{-\xi^2 \Omega_p}{8}\right) \mathcal{D}_{-2}\left(j \xi \sqrt{\frac{\Omega_p}{2}}\right) \exp\left(\frac{-\xi^2 N_0}{4}\right) \]  

(A.16)

The received PDF can be obtained by taking the inverse Fourier transform of \( F_R(j\xi) \).

Substituting Eq.(A.16) in Eq.(A.8) gives

\[ f_R(r \mid \varphi) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp\left[-\xi^2 \frac{N_0}{4} - \frac{\xi^2 \Omega_p}{8} + j\xi r\right] \mathcal{D}_{-2}\left(j \xi \sqrt{\frac{\Omega_p}{2}}\right) d\xi \]

(A.17)

\[ = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp\left[-\xi^2 \frac{(2N_0 + \Omega_p)}{8} + j\xi r\right] \mathcal{D}_{-2}\left(j \xi \sqrt{\frac{\Omega_p}{2}}\right) d\xi \]

It follows from the above expression that the PDF \( f_R(r \mid \varphi) \) is a function of the noise-variance \( \frac{N_0}{2} \) and parameter \( \Omega_p \) of Rayleigh PDF.

The closed form solution to Eq.(A.17) can be evaluated by using the following result [59] (p.890)

\[ \int_{-\infty}^{\infty} e^{ijy-\frac{(1+i\lambda)^2}{4}} \mathcal{D}_\nu [x \sqrt{1 - \lambda}] dx = \sqrt{2\pi} \lambda^{\nu/2} e^{-\frac{(1+i\lambda)^2}{4}} \mathcal{D}_\nu \left[jy\sqrt{\lambda^{-1} - 1}\right] \]

(A.18)

\[ Re[\lambda] > 0 \]

Comparing Eq.(A.17) and (A.18)

\[ \nu = -2, \quad y = r \]

(A.19)

The arguments of the parabolic-cylinder function and the exponent of \( e \) gives two equations for the value of \( \lambda \)

\[ 1 + \lambda = N_0 + \frac{\Omega_p}{2} \]

\[ \sqrt{1 - \lambda} = j\sqrt{\frac{\Omega_p}{2}} \]

(A.20)
Solving the above equations, \( \lambda \) is given by

\[
\lambda = \frac{2 + \Omega_p}{2} \quad \text{iff} \quad \frac{N_0}{2} = 1
\]  

(A.21)

Therefore, the closed form solution of \( f_R(r \mid \varphi) \) integral exists only for the special case when \( \frac{N_0}{2} = 1 \). Substituting the value of \( \lambda \) the received PDF is

\[
f_R(r \mid \varphi) = \frac{1}{2\pi} \left( \sqrt{2\pi} \frac{2}{2 + \Omega_p} e^{-\frac{(4 + \Omega_p)^2}{4(2 + \Omega_p)}} \mathcal{D}_{-2} \left[ -r \sqrt{\frac{\Omega_p}{2 + \Omega_p}} \right] \right)
\]  

(A.22)

Further simplification can be obtained by using the following result in [59] (p.1095)

\[
\mathcal{D}_{-2}(\psi) = e^{\frac{\psi^2}{2}} \sqrt{\frac{\pi}{2}} e^{-\frac{\psi^2}{2}} \left( \Phi \left( \frac{\psi}{\sqrt{2}} \right) - \psi \left( 1 - \Phi \left( \frac{\psi}{\sqrt{2}} \right) \right) \right)
\]  

(A.23)

where, \( \Phi \) is the error function defined as [59] (p.938)

\[
\Phi(\psi) = \frac{2}{\sqrt{\pi}} \int_0^\psi e^{-t^2} dt
\]  

(A.24)

Substituting Eq.(A.23) in (A.22)

\[
f_R(r \mid \varphi) = \frac{1}{2 + \Omega_p} e^{-\frac{(4 + \Omega_p)^2}{4(2 + \Omega_p)}} B(\psi)
\]  

(A.25)

where \( B(\psi) = \sqrt{\frac{\pi}{2}} e^{-\frac{\psi^2}{2}} - \psi \left( 1 - \Phi \left( \frac{\psi}{\sqrt{2}} \right) \right) \), \( \psi = -r \sqrt{\frac{\Omega_p}{2 + \Omega_p}} \)

The received PDF \( f(r \mid \varphi) \) is given by the integral in Eq.(A.17). The closed form solution of the integral exists only for the special case when \( \frac{N_0}{2} = 1 \) and is given by Eq.(A.25).
APPENDIX B

JAKE’S SIMULATOR

From Eq.(2.10), the low-pass received signal at time $t$ is

$$
 r(t) = \sum_n a_n s(t - \tau_n) e^{-j2\pi \phi_n(t)} 
$$  \hspace{1cm} (B.1)

where,

$$
 \phi_n(t) = [f_c + D_n] \tau_n + t D_n 
$$  \hspace{1cm} (B.2)

To generate a Rayleigh distributed trace $r(t)$ using simulation, the summation in the above equation is carried out over $n = N$ incoming waves. These waves are generated by using $N$ oscillators. This approach to generate Rayleigh distributed signal was proposed by Jakes [6]. The Jake’s simulator is implemented with a small modification in this work. The details are explained next.

Let the local mean power of the signal $s(t)$ be $E_0$ at the location where $r(t)$ is observed. Assuming equal strength multipath components, each path contributes the power $\frac{E_0}{N}$. Therefore, the first term in Eq.(B.1) is

$$
 a_n s(t - \tau_n) = \sqrt{\frac{E_0}{N}} 
$$  \hspace{1cm} (B.3)

Let $\cos \beta_n$ be the angle between the incident ray and the mobile velocity vector. Substituting $D_n = \cos \beta_n$ from Eq.(2.5) for phase given by Eq.(B.2), the signal $r(t)$
can be written as

\[ r(t) = \sqrt{\frac{E_0}{N}} \sum_{n=1}^{N} e^{-j[\phi_n + 2\pi f_m t \cos(\beta_n)]} \]  

(B.4)

where \( \phi_n = 2\pi [f_c + D_n] \tau_n \). The inphase and quadrature components of \( r(t) \) can be obtained as

\[ r_I(t) = \text{Re}[r(t)] \quad r_Q(t) = \text{Im}[r(t)] \]  

(B.5)

where \( \text{Re}[\cdot] \) represents the real part and \( \text{Im}[\cdot] \) represents the imaginary part of the argument. To evaluate \( r(t) \), the angle \( \beta_n \) between the incident ray and the mobile velocity vector is assumed equi-spaced in the interval \([-\pi, \pi] \). Therefore \( \beta_n \) can be written as

\[ \beta_n = \frac{2\pi n}{N}, \quad t = 0 \]  

(B.6)

The determination of phase \( \hat{\phi}(t) \) is slightly modified in this work as compared to the traditional Jakes simulator. The traditional Jake’s simulator [2] assumes equi-spaced phases without using any time dependence. Whereas, we have modeled \( \hat{\phi}_n \) randomly varying with time. This is done to take into account uniformly distributed phase in the Rayleigh pdf. The phase \( \hat{\phi}_n \) is modeled as a first order AR process given by the following equation

\[ \hat{\phi}_n(t) = A \hat{\phi}_n(t-1) + \eta(t), \quad t > 0 \]  

(B.7)

The initial value \( \hat{\phi}_n(t), t = 0 \) is chosen equi-spaced between \([-\pi, \pi] \) that is,

\[ \hat{\phi}_n(t) = \frac{2\pi n}{N}, \quad t = 0 \]  

(B.8)

Assuming that the oscillators do not abruptly change phases, \( \hat{\phi}_n(t) \) should be heavily correlated in time. Therefore the value of regression coefficient \( A \) should be chosen
close to 1. The regression coefficient $A$ can be adjusted in the range $0 < A < 1$ to generate less correlated or uncorrelated fast fading envelope. To generate the Bessel correlation function, $A = 0.999$ gives satisfactory results. The noise $\eta(t)$ has distribution $N(0, \sigma_N^2)$ where $\sigma_N^2 = (1.0 - A^2)\frac{\sigma^2}{4}$. The number of oscillators used in this work $N = 65$ to generate the reference traces for all the simulation experiments.
BIBLIOGRAPHY


