Channel Assignment for Time-Varying Demand

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Abstract

This paper presents an integer programming model for dynamic channel assignment (DCA) under space and time-varying traffic demand. The algorithm minimizes the number of channels required to satisfy the traffic demand using a threshold based decision criteria on the carrier-to-interference ratio. A neighborhood based search procedure uses the most recent channel state information to perform a feasible assignment when the demand changes. This technique accelerates the convergence of the algorithm to a local minima and allows an evaluation of channel gains obtained with increasing neighborhood sizes. This procedure will also minimize the number of channel reassignments in cells whose demand is time-invariant. The performance of the channel assignment algorithm (SA) is examined relative to the spatial distribution of the cells with time-varying demand. Channel gains obtained with DCA relative to the SA scheme range from $30-40\%$ for the examples discussed.

1 Introduction

Wireless cellular networks are bandwidth and power limited. A finite frequency spectrum is available for provision of commercial services. Based on the service requirements, the allocated spectrum is divided into a number of channels. Channels are assigned to geographical regions based on expected traffic demand. The reuse of frequencies at distances large enough to minimize co-channel interference is a basic design principle in cellular networks. Most existing networks have a fixed number of channels assigned (FCA) permanently to each cell for its exclusive use. This arrangement is inefficient for wireless transmission of packet voice, video and data services. The support of these multimedia applications is the focus of third and fourth generation wireless systems. The traffic patterns that characterize packet data and video have been found to exhibit high temporal variability [6, 5]. In particular, such packet traffic exhibit persistence in both underload and overload states for durations longer than that predicted by classical negative exponential distributions. This state-of-affairs will lead to both under utilization of resources and higher blocking with FCA schemes for bursty traffic sources. The requirement of larger channel bandwidths for broadband transmission will also impose a stringer requirement on optimal allocation of channel resources. Dynamic channel assignment (DCA) schemes that can perform at the time-scale of traffic variation are therefore an important component in future wireless networks. Channel assignment schemes that minimize the overhead involved in reassignment of existing calls, while maximizing channel utilization are of particular interest.

A comprehensive survey of fixed, dynamic and hybrid channel assignment schemes is provided by

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Katzela et. al. [7]. The simplest modifications to FCA are based on borrowing channels from the richest neighboring cells to minimize future call blocking probability. Anderson [1] discusses simulation studies of these algorithms and shows that the number of search steps required can limit the performance of the approach. Modifications to reduce the number of search steps have also been considered [2]. This involves channel ordering schemes where the fixed-to-borrowable channel ratio is dynamically varied according to changing traffic conditions.

In DCA there is no fixed relationship between channels and cells and all channels are available for assignment to all cells. Channel assignment takes place through minimization of a cost function such as the allocated bandwidth under the constraint that channel reuse takes place above specified interference levels. Many algorithmic approaches have been proposed for achieving the objective function subject to the set constraints. Murphey et al. [4] provide a comprehensive survey of these algorithms. They may be broadly classified as graph theoretic, meta-heuristic search based techniques and mathematical programming approaches. In the graph-theoretic abstraction of the problem each transmitter is represented by a node in a graph and two transmitters share a graph edge if using the same channel could create interference. Graph coloring approaches and Integer programming (IP) formulations have been used to tackle this graph problem [8]. Capone et. al. [3] propose a tabu search to determine the solution that is beyond locally optimal. This meta-heuristic search technique for exploring the solution space relies heavily on the collection and management of search history data to minimize reexploration of unproductive parts of the search space. Tabu search is not guaranteed to produce an optimal solution and there are not, in general, any guarantees about the closeness of its solution to the optimal value. This is in contrast to mathematical programming techniques and is also in contrast to approximation algorithms, which guarantee that their solutions are within some known factor of optimal. Heuristics can also be used to provide starting bounds for mathematical programs resulting in hybrid optimization approaches. Smith et. al. [9] use neural-networks, simulated annealing and steepest descent heuristics to solve a nonlinear integer-programming (IP) formulation of the assignment problem.

Although a variety of algorithmic models and solution techniques have been proposed for DCA, the performance of such algorithms in the context of varying traffic demand has been examined to a lesser extent. Argyropulos [10] et. al. showed that in the presence of spatial load-imbalance DCA produced much greater improvements in the performance than FCA relative to gains for uniform load distribution. Everitt and Mansfield [11] assumed that the mean traffic in each cell is Gaussian distributed and showed the DCA is more resilient to traffic volatility than FCA. In this paper an integer programming model for a centralized frequency assignment problem is proposed. The objective is to minimize the number of channels used. The assignment is to satisfy a time-varying demand where the demand for a given transmitter/cell is often larger than one. The distance between transmitters is used to control co-channel interference. Two transmitters are assigned the same channel only if there is sufficient distance between the transmitters. The selection of constraints for channel reuse is based on the C/I ratio for a given frequency, where C is signal to be acquired and I represents sum of interfering signals. This IP model will be used to perform channel assignments with temporal variation in the demand. For a grid of square cells, a time-step simulation is used that invokes a traffic demand model for each time step. The traffic demand is fed as input to a channel assignment model that seeks the smallest number of channels satisfying the traffic demand for that time step.

Section 2 discusses the demand models; these are based on discrete-time, finite state Markov chains. Section 3 presents the channel assignment model; this is based on integer programming. Section 4 presents results of dynamic channel assignment. Section 5 concludes the paper.

2 Traffic Demand Model

The influence of bursty traffic sources on dynamic channel allocation will be examined in relation to their spatial distribution and temporal variation. Each cell is assumed to generate either constant bit rate (CBR) of C demand units per second or a variable bit rate (VBR) demand where transition from C to V demand units occur randomly in time. It is assumed that in the starting state, each cell is assigned C channels. The channel holding time is one time unit. Overload conditions occur when one or more sources are in the VBR state V. For packet data, a limited queueing space is often available to facilitate channel search, allocation and setup procedures. A VBR cell that persists in the overload state for a finite time therefore demands an increasing number of channels in each successive time step. This is the traffic scenario considered in this study.

The demand generated by each cell is independent of the traffic in other cells. VBR traffic is modeled using a two-state discrete time Markov chain. This traffic model is easily extended to larger number of states. Markov chains afford a tractable model for controlling the level of temporal correlation in the traffic. The correlation measure used in this work is the expected time a source remains in the over load state. If the probability of transition from V to C demand units is β, the expected duration in the overload state is $T_{OL} = \beta^{-1}$. Dynamic channel allocation will be examined for an increasing range of $T_{OL} \ : 1, 2...10$.

The minimum channel requirement is also highly dependent on the spatial distribution of the VBR sources. To demonstrate this effect three representative spatial distributions R_1, R_2 and R_3 are overlaid on a 7×7 grid of square cells as shown in Figs. 1(a-c). The ratio of the number of VBR to CBR cells is fixed at 10%. The channel requirements with time are a function of the number of VBR cells that exist within the frequency reuse distance. R_1 and R_2 represent extreme cases of deterministic spatial configuration where the distance between VBR type cells is maximum and minimum respectively, relative to the grid size. R_3 is an example of a randomly distributed configuration. The performance of the IP model described in Section 3 will be examined for these three cases subject to temporal variation in demand. First as a baseline comparison for resource utilization the number of channels is estimated using an effective bandwidth calculation of the Markov sources.

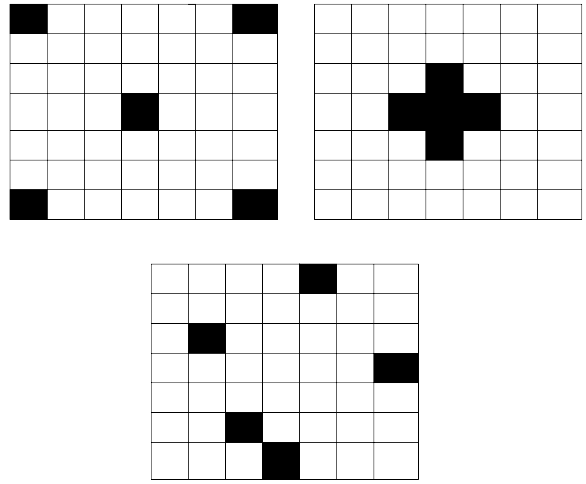

Figure 1: (a-c)Spatial distribution of VBR sources

2.1 Effective Bandwidth Based Assignment for Markov Demand

The capacity requirements of independent Markov sources multiplexed on a common set of resources can be estimated using an effective bandwith approximation. Let Q_m denote the infinitesimal Markov generator of m identical multiplexed sources. This can be determined from the $M - fold$ Kronecker sum of the single source generator Q. Let R_m denote the diagonal source rate matrix with elements $r_i, \quad i = 1, 2...m$ representing the state dependent channel demand. The serving capacity is C. Under the assumption that the asymptotic decay rate of the complementary distribution of the number waiting n_w in the queue $Pr(N_w > B) \ \approx e^{-s \ n_w}$, the capacity required for a specified performance constraints $[Pr(N_w > B), s]$ may be derived as the maximal real eigenvalue of the matrix $\left[\frac{Q_m}{s} \ + \ R_m\right]$. The single source modeled by a two state DTMC is assigned demand rates $r_1 \ = \ 1$ and $r_2 \ = \ 2$ respectively. The channel capacity required for $m = 1, 2, 3$ multiplexed sources to satisfy performance constraints

Effective Bandwidth Based Allocation			
T_{OL}	R_1	R_2	R_3
2	11	19	15
3	12	24	18
4	13	29	21
5	14	34	24

Table 1: Requirements under FCA

$B = 1, s = 10^{-3}$ was computed. These values are represented by $C_{vbr}(m)$. The channels required for the three spatial configurations in Fig. 1 were determined as $C(T_{OL}) = C_{cbr} + \sum_{m=1}^{3} NC_{vbr}(m) * C_{vbr}(m)$, where $C_{cbr} = 9$ is fixed as the requirement for the CBR cells for a reuse distance of two, $NC_{vbr}(m)$ represents the number of VBR clusters of size m that reside within the specified reuse distance. The reuse distance considered here is consistent with the C/I interference threshold used in the IP model. The results are given in Table I. The efficiency of DCA will be evaluated relative to these FCA assignments.

3 Channel Assignment Model

The traffic demand for a given simulation time step t is used to formulate the channel assignment problem as an IP model. The overall approach is first described, followed by the specifics of the IP model.

3.1 Approach

$IP(t)$ denotes the minimum number of channels found by the assignment model for time step t. The assignment generated by time step $t-1$ is used to influence the assignment approach for t in the following way. For each $cell(i,j)$ in the grid a neighborhood of size s: $N_s(i,j) = \{cell(k,l) \; |i-k| \leq s, |j-l| \leq s\}$ is defined. For time t, a sequence of IP models are solved in which the neighborhood size increases from one model to the next. Let $D(i,j,t)$ denote the demand for $cell(i,j)$ at time t. For a given neighborhood size, $D(i,j,t)$ is compared with $D(i,j,t-1)$. If the demand has changed, the assignments for cells in $N_s(i,j)$ are allowed to change at time t. Cells that are not in a neighborhood of any cell whose demand changes from time $t-1$ to time t keep their time $t-1$ assignments at time t. As the neighborhood size increases, this strategy has the effect of progressively relaxing constraints on the assignment solution space. Once the neighborhood size equals the size of the grid, the assignments of all cells are allowed to change. If we let $IP_n(t)$ denote the minimum number of channels that can be found by the assignment model at time t for a neighborhood of size n, then an increasing sequence of h neighborhood sizes imposes the following neighborhood constraint on the solutions: $IP_0(t) \geq IP_1(t) \geq ... \geq IP_h(t)$

This iterative neighborhood approach is used for two reasons: 1) to control the running time of the IP model and 2) to allow investigation into how sensitive a global minimum is to local demand changes. The latter is of interest when comparing centralized DCA with distributed/localized DCA approaches.

3.2 IP Model

A brief description of a typical minimum order frequency assignment IP formulation for static demand is provided. This is followed by a description of the approach proposed for demand that changes in time.

3.2.1 Typical Static Demand Formulation

Minimum order frequency assignment for static demand typically has two groups of binary decision variables: 1) variables that show if a given transmitter is assigned to a given frequency, and 2) variables that reflect whether or not a given frequency is used in the solution. The objective function of the canonical formulation minimizes the sum of the type (2) variables. In the standard formulation, type (2) variables are linked to type (1) variables via a set of constraints.

The notations used in the algorithm are as follows:
t : time and $cell(i,j)$:cell at row i and column j.
The parameters and precalculated values are:
r = number of rows in the cell grid
c = number of columns in the cell grid
q = number of frequencies
B = interference threshold value

e = tolerance for small values

α = path-loss exponent

$D(i,j,t)$ = traffic demand at time t for $cell(i,j)$. This is calculated as described in Section 2.0.

$d(i,j,k,l)$ = Euclidean distance from $cell(i,j)$ to $cell(k,l)$.

$S(i,j,k,l)$ = strength of the signal received due to transmission between $cell(i,j)$ and $cell(k,l)$.

The co-channel interference between $cell(i,j)$ and $cell(k,l)$ is calculated as:

$$S(i,j,k,l) = \frac{1}{d(i,j,k,l)^{\alpha}} \tag{1}$$

The integer binary decision variables are:

$$A(i,j,f) = \begin{cases} 1 & if\ frequency\ f\ assigned\ cell(i,j) \\ 0 & otherwise \end{cases} \tag{2}$$

$$a(f) = \begin{cases} 1 & if \sum_{i,j} A(i,j,f) \geq 1 \\ 0 & otherwise \end{cases} \tag{3}$$

The model can be stated as follows:

$$min \sum_{f=1}^{q} a(f) \tag{4}$$

subject to:

$$(q)a(f) - \sum_{i,j} A(i,j,f) \geq 0$$

for $1 \leq f \leq q, 1 \leq i \leq r, 1 \leq j \leq c$ (5)

$$\sum_{f=1}^{q} A(i,j,f) \geq D(i,j,t) \quad for\ 1 \leq i \leq r, \quad 1 \leq j \leq c \tag{6}$$

$$\left[\frac{S(i,j,i,j)}{B} - A(i,j,f)\right] \sum_{(k,l)\neq(i,j)} S(i,j,k,l) \ -$$

$$\left[\frac{S(i,j,i,j)}{B} A(i,j,f)\right] \geq \sum_{(k,l)\neq(i,j)} S(i,j,k,l)A(k,l,f)$$

for $1 \leq f \leq q, \quad 1 \leq i,k \leq r, \quad 1 \leq j,l \leq c$ (7)

The first block of constraints in Eq. (6) links the binary variables $A(i,j,f)$ that show if a given transmitter is assigned to a given frequency to the binary variables $a(f)$ that reflect whether or not a given frequency is used in the solution. There are qrc of the first type (assignment) variables, q of the second type, and q linking constraints. The second block of constraints in Eq. (8) is the set of demand constraints. There are rc of these constraints. The third block in Eq. (7) contains frequency-distance interference constraints. There are qrc of these constraints. Note that all of the constraints are linear. The third block of constraints is a linearization of a set of nonlinear constraints. Since one transmitter is assumed per region and rc is the number of regions, the number of variables and constraints is proportional to the product of the number of frequencies times the number of transmitters. This is consistent with IP model sizes surveyed in [4].

However, since the size of the model is a function of the number of channels, one challenge of IP channel assignment, as noted in [4] is to provide a reasonable initial upper bound on the number of channels. The model for time t is clearly feasible if the initial upper bound is equal to: $\sum_{i,j} D(i,j,t)$. However, if the upper bound is too large the size of the model may make it difficult for an IP solver to provide a timely solution. On the other hand, if the bound is too small the model becomes infeasible. A greedy sequential assignment (SA) algorithm is used to provide an initial upper bound on the number of channels that is not necessarily optimal. The algorithm is based on a sequential and fair assignment strategy that uses the same frequency-distance interference conditions as the IP model. The SA scheme traverses the cells in a sequential order assigning one channel to each cell subject to the interference threshold constraint. This round-robin allocation is repeated until all the cells have the required demand.

3.2.2 Neighborhood Constraints for Time-Varying Demand

For each point in time and a given number of available channels, the IP model is solved using the

Mixed Integer Programming solver component of the $CPLEX7.0^{TM}$ optimization software package. As discussed in Section 3.1, for time t the solution is obtained for a sequence of IP models of increasing neighborhood size. Cells that are not in a neighborhood of any cell whose demand changes from time $t-1$ to time t keep their time $t-1$ assignments at time t. If $cell(i,j)$ will keep its time $t-1$ assignment, an additional constraint of the form: $A(i,j,f) = v$, is included in the model, where v is the value of the time $t-1$ assignment.

4 Results

The IP model was solved for the three spatial configurations of VBR cells shown in Fig.1. In cases where the number of channels corresponding to the best integer solution remained constant over a large number of nodes in the Branch and Bound search, the search was terminated early. In these cases the solution is not necessarily optimal so the neighborhood constraint on the solutions is not necessarily enforced. For each spatial configuration, the demand generated by the marked cells increased at a uniform rate in time. The number of channels required at this sustained demand rate was determined in each time step using the neighborhood based restriction discussed above. The channel requirement $IP_n(t)$ was obtained for $t = 1, 2..10$ for $n = 0, 1, 2$. The largest neighborhood size for a cell is given by the minimum distance to another VBR cell.

Figs. 2(a-c) depict the solutions obtained. For each case, the upper-bound solution of the sequential assignment SA algorithm is compared with the best IP solution across neighborhoods and that obtained with neighborhood size $n = 0$ denoted IP_0.

The demand service rate was set equal to C units per unit time interval. The path-loss exponent $\alpha = 3.5$ and the CIR threshold value $B = 27234$, results in reuse distances greater than or equal to two cells. The traffic dispersion due to VBR sources that result in overload conditions is $\frac{V}{C} = 2$. The rate of increase in number of channels required is a function of the average size of the CBR neighborhood around a VBR cell and the reuse distance allowed by the CIR constraint. For CBR neighborhoods greater than the reuse distance, the increase in required channel capacity is simply the excess arrival rate $\lambda = 1$ in one VBR cell. This is one channel for the example in Fig. 2(a). All three solutions exhibit this expected trend. The IP result is seen to converge to the local neighborhood based solution for large t. In case R_2 a VBR cluster size $V_{cl} = 3$ exists within the reuse distance. Therefore the solutions follow the expected increase of $V_{cl} \times \lambda = 3$. The configuration in R_3 having a cluster size of $V_{cl} = 2$ also exhibits the expected channel requirement rate of 2 channels per time unit.

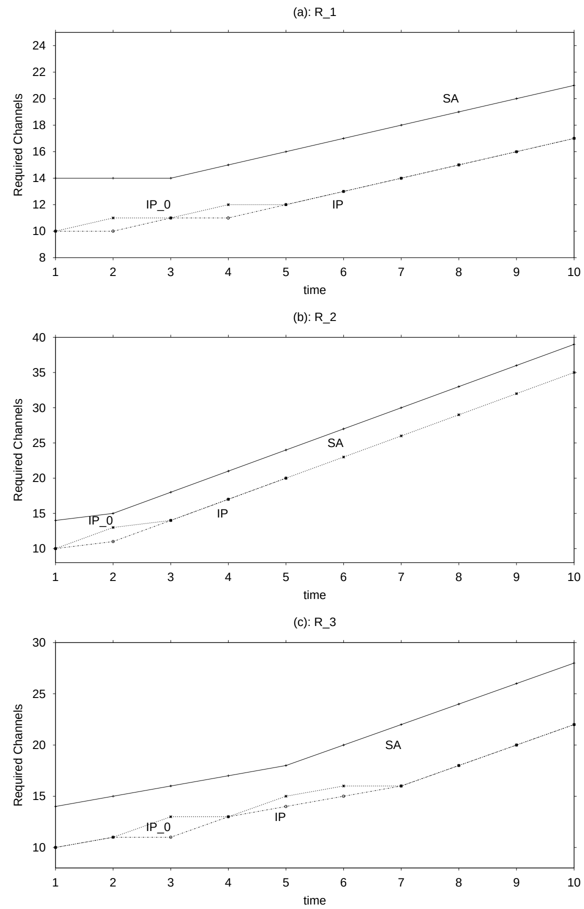

Figure 2: Channel requirements using DCA

In comparison to SA schemes, IP models are seen to provide channel gains ranging from $30-40\%$. Relative to FCA assignments using effective bandwidth estimates, the channel gains range from $15-40\%$, the larger gains result for spatial configurations with VBR clusters. The presence of two distinct trends for small and large t occurs due to the enforcement of the frequency reuse constraints. The slower rate of increase in channel requirements for small time results from the existence of assigned frequencies in CBR cells that are reused in VBR cells when their demand increases. There is an upper limit to the number of such available frequencies, that depends on the grid size. When the demand exceeds this reuse availability for large time, new channels demanded are assigned exclusively in the VBR cells. This is also the reason why the IP and IP_0 solutions exhibit differences in the small time regime. The reassignment of existing frequencies is achieved only when the neighborhood size is increased beyond zero.

In any dynamic assignment scheme a significant overhead results when existing assignments have to be inter-changed to enable optimal assignments. The improvement achieved relative to a localized neighborhood constraint must be evaluated to determine if the maximum packing algorithms are warranted. In the examples considered, the efficiency of maximal packing is seen to be improved by less than 10%. This feature is however a function of the spatial distribution of the VBR sources.

5 Conclusions

The computational difficulty of channel assignment for large problems makes it difficult to obtain optimal results. Based on this, one might intuitively expect minimizing the number of reassignments and minimizing the number of channels to be competing goals requiring a tradeoff. While this might actually be true for optimal solutions, our work suggests that exploiting locality of demand changes can allow these goals to be synergistic for dynamic channel assignment. The dynamic channel assignment algorithm proposed shows that the achievable channel gains depend on the spatial configuration of the cells with time-varying demands. In particular, the solutions can be reasonably well predicted using geomtric descriptions of the spatial distribution. The results suggest that the monitoring and measurement of traffic descriptors based on space and time locality can increase the efficiency of implementation of mathematical programming algorithms.

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