Comparing Channel Assignment Results from IP Algorithm and DCA Heuristic

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Abstract

This note accompanies the DCA algorithm paper [1] by Daniels et. al. This paper presents two channel assignment algorithms, one based on an Integer Programming (IP) model and the other based on a heuristic. The number of channels required to satisfy linearly increasing demand function in the randomly distributed $Type_v$ cells are presented here for reuse distances greater than 2 and 3. The results show that the new heuristic closely matches the IP solution in the number of channels required under zero blocking condition. The DCA heuristic is also applied to evaluate the blocking probabilities for demand generated by a two state Markov chain based arrival process and uniformly distributed holding times.

1 Linearly Increasing Demand

The DCA heuristic is compared to the IP strategy by considering the linearly increasing demand function of $Type_v$ cells. The results of the DCA heuristic are shown in Figure 2 under the label H_t and may be compared to the IP solution denoted by U_{c_t}. These results are for the case where reuse distance is greater than 2 and the threshold that satisifies this constraint is $B = 27234$. The number of channels found required by the heuristic is seen to be comparable to that generated by the IP algorithm. Typically, the heuristic matches the IP solution to within one channel, and deviates by at most two channels in cases (e,f,g). For case (c), the heuristic improves on the IP performance by requiring one less channel. The performance may also be compared with respect to the average channel reuse afforded by the two approaches. The heuristic reuses each channel on average 4.64 times, in comparison with the IP which reuses each channel an average of 4.5 times. The policy of reusing a channel at a location that creates a median change in residual interference was compared to two other policies where the location is selected so as to minimize or maximize the change in residual interference. The latter policy is often referred to as maximum packing. The median policy was typically the better choice, particularly in cases where the $Type_v$ cells were spread across the entire spatial grid. Although the minimum policy was often comparable to the median solution, this approach can lead to divergent solutions in cases such as (d,f), where large $Type_v$ clusters reduce the probability of finding a location that is furthest from a group of interfering cells. The maximum packing policy is in all cases the worst performing policy since the cumulative interference quickly constrains the channel reuse factor. The heuristic's execution time is observed to increase linearly as a function of demand, which is consistent with the analysis of the algorithm [1]. The heuristic is fast, with typical running time of .002 seconds per unit of demand on a 600 MHz Compaq Alpha server.

A second set of results is shown in Figure 3 where reuse distance is greater than 3 and the threshold $B = 125000$. Calculation of the threshold to satisfy the reuse requirements, given a channel model is described in an accompanying note.

2 Markovian Demand Model

The application of the DCA heuristic based algorithm is examined next in the context of blocking probabilities generated for randomly varying demand in time. $Type_c$ cells generate a constant demand of $D_c = 1$ channel in each time unit. $Type_v$ cells generate demands using a two state (on-off) discrete-time Markov chain model with uniformly distributed holding times. In the on and off states the cell demand rates are D_{on} and D_{off} respectively, with $D_{on} > D_{off}$. The channel holding times in the on and off states are characterized by independent identically distributed random variables with an average of $\hat{\tau}$ time units.

The expected demand per $Type_v$ cell is obtained using the sum of the average on and off periods of the Markov chain as a reference cycle duration. Let $\hat{T}_{on}$ and $\hat{T}_{off}$ denote the expected durations of the arrival process in the on and off periods respectively. Note that $\hat{T}_{on} = \frac{1}{p_{10}}$ and $\hat{T}_{off} = \frac{1}{p_{01}}$. The average holding times may in general be different in the on and off periods and are denoted as $\hat{\tau}_{on}$ and $\hat{\tau}_{off}$ respectively.

The average channel demand per $Type_v$ cell can be shown to be obtained by the relation,

$$\hat{D} \;=\; \frac{(D_{off}\ \hat{\tau}_{off}\ \hat{T}_{off}) + (D_{on}\ \hat{\tau}_{on}\ \hat{T}_{on})}{\hat{T}_{on} + \hat{T}_{off}} \tag{1}$$

The blocking performance is examined for the case where $D_{on} = 2$, $D_{off} = 1$ and the channel holding times are discrete equiprobable values ranging from 1 to 5 unit time durations. The average holding times in the on and off states are $\hat{\tau}_{off} = \hat{\tau}_{on} = 3$. These parameters bound the per $Type_v$ cell demand variation between values of 1 and 10 respectively in each time interval. Selecting the Markov parameters as, $p_{00} = 0.55,\ p_{01} = 0.45,\ p_{10} = 0.2,\ p_{11} = 0.8$, the average channel demand rate per $Type_v$ cell is given by Eqn. 1 as 5.1 channels.

The blocking probabilities were simulated by providing a fixed number of channels C and applying the DCA heuristic to perform the assignment in each time step of the Markov process evolution. The blocking probabilities plotted in Figure 4 (a-i) for each of the nine ensembles given in Figure 1 are obtained by counting the number of blocked calls in simulation lengths of $30,000$ time steps. The results are for reuse distance greater than 2. The results in each panel demonstrate the effect of choosing assignments based on the three different cost metrics in the DCA heuristic discussed above. Probabilities denoted as HP_1 depict the case where channels were assigned to positions that resulted in a median change in interference, HP_2 denotes the case when the change in interference was minimized (channels placed furthest from existing assignments) and HP_3 show the results when channel reuse was subjected to the maximum packing condition. It is seen that the maximum packing constraint demonstrates the worst performance and the median interference change proves to be the better policy in almost all cases. We note that the maximum packing constraint has been applied as a candidate in several DCA algorithms [2, 3] for evaluating how a good dynamic channel assignment algorithm will perform. Our results demonstrate that when cumulative interference is a constraint, the maximum packing policy is not a good indicator of DCA performance.

To compare the blocking probabilities across the nine different ensembles, the unique spatial features of the $Type_v$ cells must be taken into account. This can be achieved by considering the IP solutions U_{c_t} or heuristic solutions H_t obtained, for that time instant when all $Type_v$ cells assume the same demand value of $\hat{D}$. For the case analyzed here, $\hat{D} = 5.1$ and $16 \leq H_t[n] \leq 25$ across the nine spatial ensembles, $n : [a-i]$. Denoting $H_t[n]$ under the constraint of fixed t as, C_{avg}, the effective system load may be represented as $\gamma = \frac{C_{avg}}{C}$. The solution C_{avg} is unique to each spatial distribution and it effectively captures both spatial and average temporal dynamics of the network demand.

Figure 5 depicts the blocking probabilities for policy HP_1 of all nine ensembles as a function of γ. The relative invariance in the structure of the individual blocking probabilities when represented with respect to γ indicates that C_{avg} captures the necessary differences among the spatial distributions and can be applied as a suitable renormalization factor. These results provide an estimate of the maximum operating

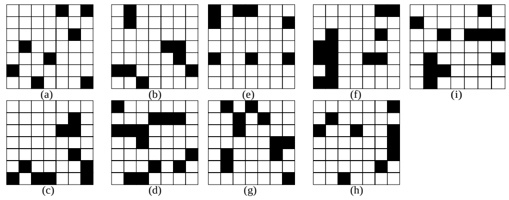

Figure 1: Stochastic spatial distribution of variable demand cells (shaded), with $Type_v$ cell parameter of 20% .

load that will maintain blocking probabilities below a specified threshold. This estimate is expected to be valid for the class of spatio-temporal distributions specified by the set of average parameters $p_v c_{max}$ and $\hat{D}$, capturing the spatial and temporal features respectively.

3 Uniform Traffic Comparison

To evaluate the benefits of dynamic channel assignment using the new heuristic, the number of channels required under fixed channel assignment considering peak demand conditions were obtained. For the case where all of the cells are assumed to be of $Type_c$, demanding the maximum of 10 channels per cell, the DCA heuristic was found to require 92 channels to satisfy the network demand. We note that 10 channels are sufficient to ensure zero blocking when all $c_{max} = 49$ cells require one channel. In the FCA case alone, the proposed heuristic has produced a savings of 8 channels. If however, one additionally considers that the peak demand arises from only the 20% fraction of $Type_v$ cells as in Figure 1, under DCA solutions U_{c_t}, the savings in the number of channels utilized, range from $51 - 71\%$.

References

[1] K. Daniels, K. Chandra, S. Liu, and S. Widhani, "Dynamic channel assignment with cumulative cochannel interference," *Submitted: Mobile Computing Review*, 2004.

[2] D.E. Everitt and D. Mansfield, "Performance Analysis of Cellular Mobile Communication Systems with Dynamic Channel Assignment," *IEEE Journal Select. Areas of Commun.*, vol. 7, no. 8, pp. 1172:1179, 1989.

[3] R.Battiti, A. Bertossi, and M. Brunato, "Cellular channel assignment: A new localized and distributed s trategy," *Mobile Networks and Applications*, 2001.

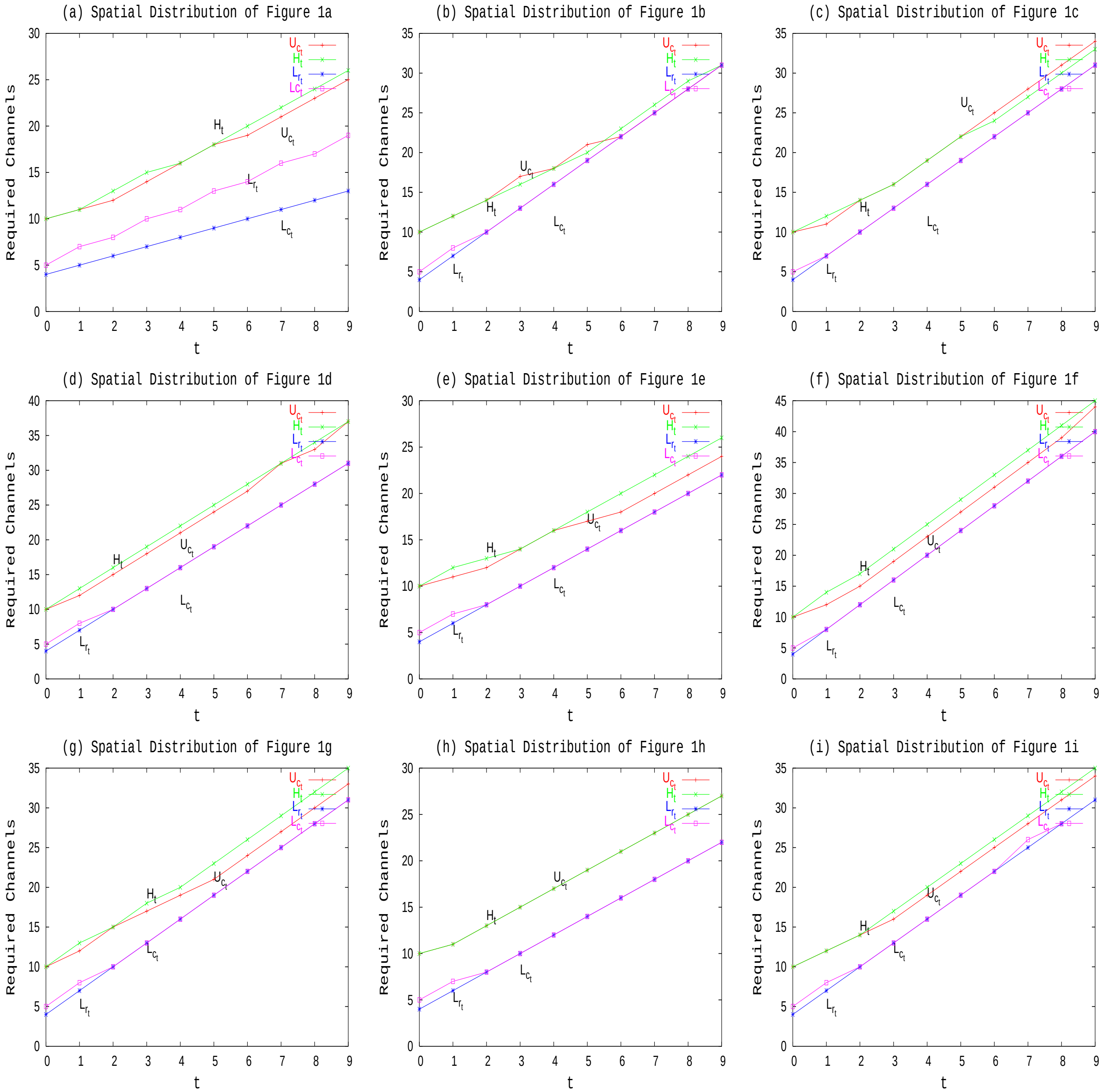

Figure 2: Minimum Number of Channels for Spatial Distributions of Figure 1 using threshold $B_2 = 27234$ for cluster size $\rho = 2$.

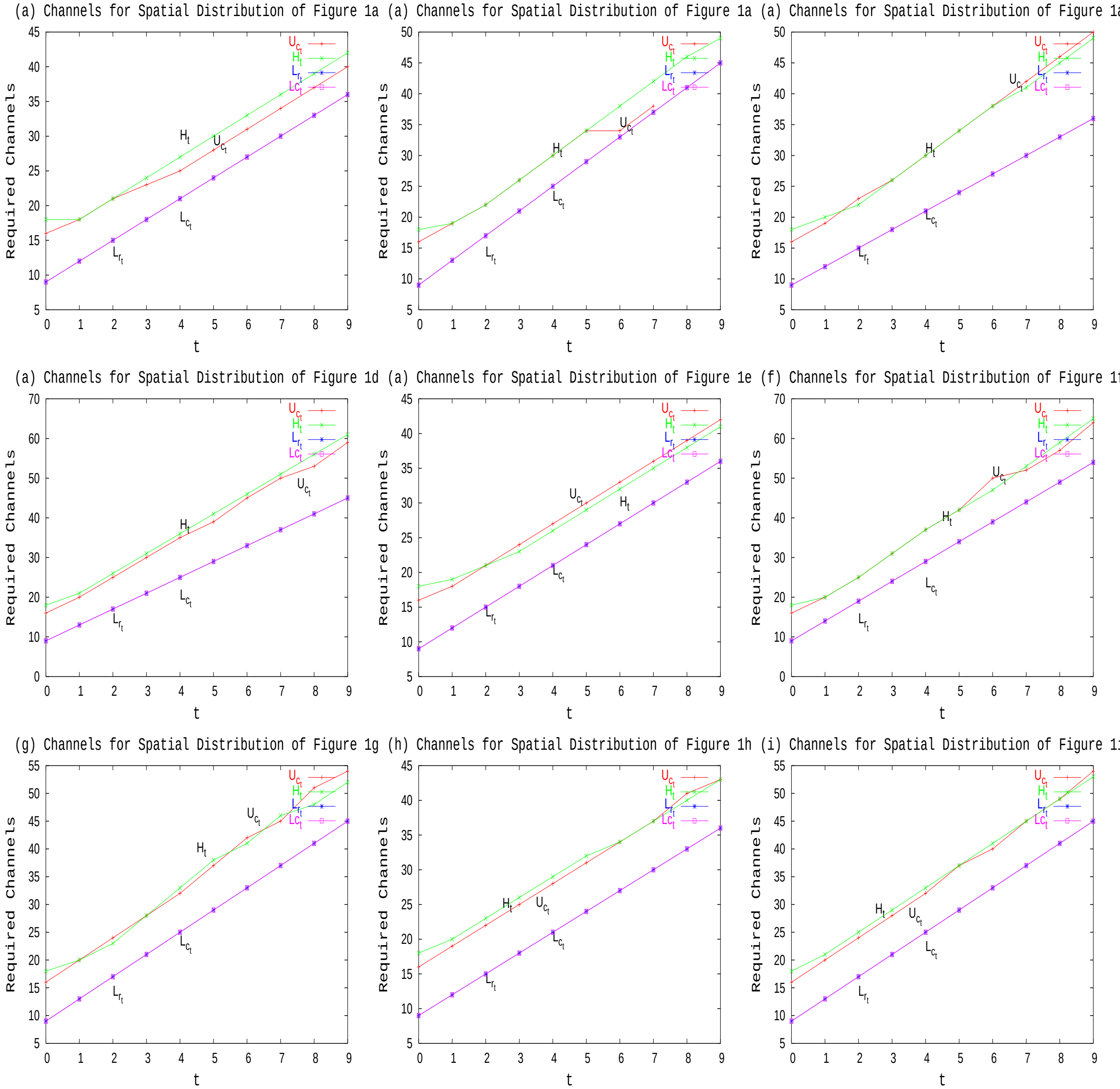

Figure 3: Minimum Number of Channels for Spatial Distributions of Figure 1 using threshold $B_3 = 125000$ for cluster size $\rho = 3$.

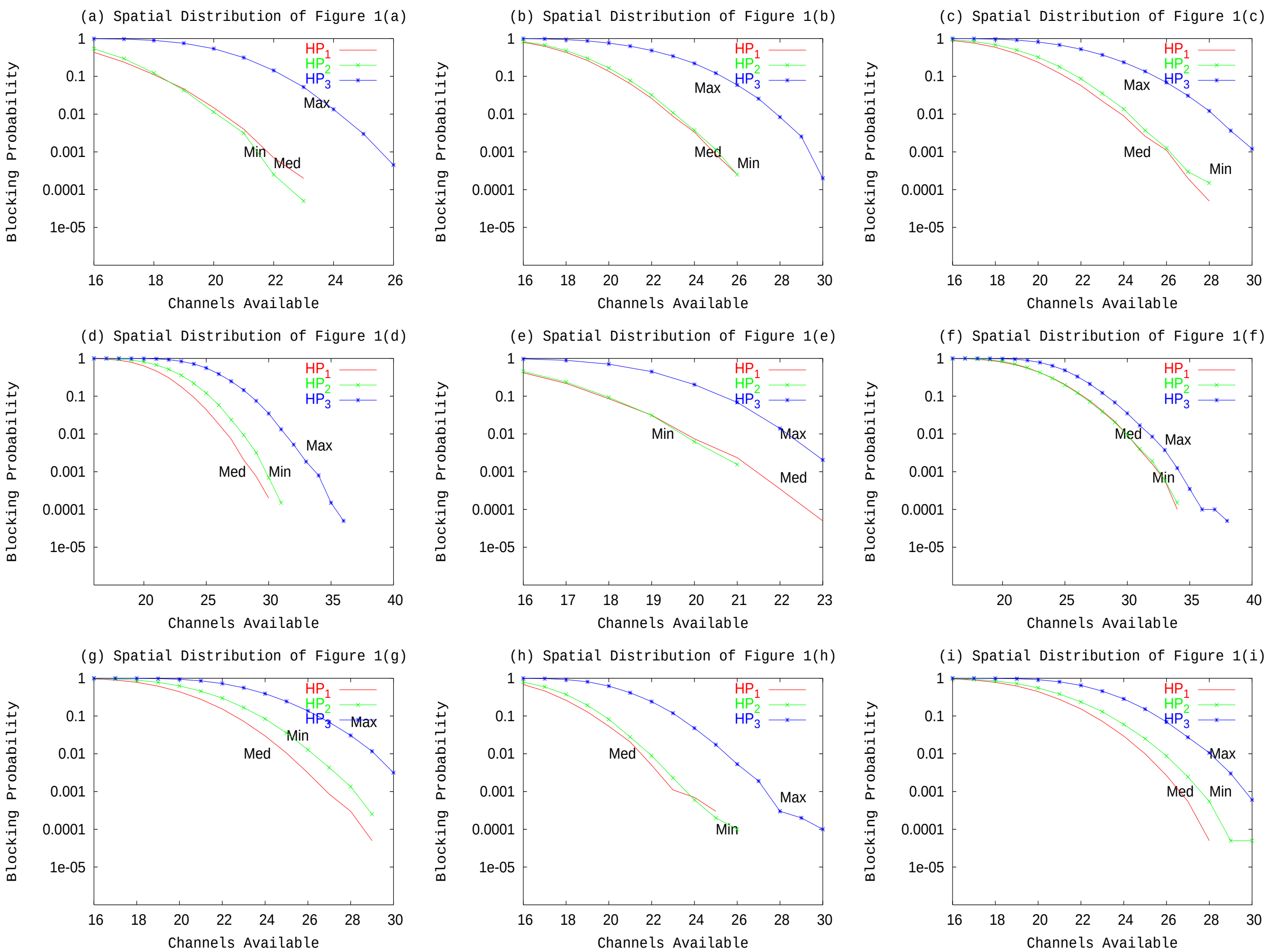

Figure 4: Blocking probabilities using DCA heuristic for on-off arrival processes and uniformly distributed holding times. (median, minimum and maximum packing policies compared)

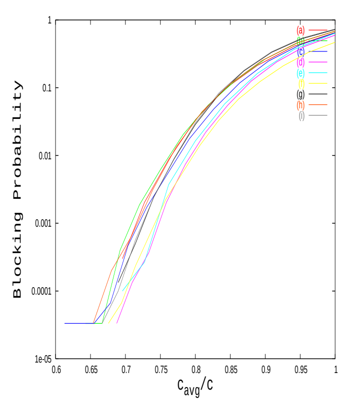

Figure 5: Blocking probabilities of all nine spatial ensembles as a function of $\gamma = C_{avg}/C$ for B_2.