Finite Population Model for Performance Evaluation Between Narrowband and Wideband Users in the Shared Radio Spectrum

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Abstract—In this paper a theoretical evaluation of channel access availability of narrow and wideband users occupying a common range of frequencies in the radio spectrum is presented. User inter-request and call duration times are taken to be random and exponentially distributed. The number of available frequency channels is fixed. The differentiating feature between narrow and wideband users is the number of frequency channels required by each user to transmit their data. The probability of a given user being blocked from acquiring a spectral channel is determined. It is shown that the degree of blocking experienced by narrowband users can only be maintained by either throttling the offered load or limiting population size of wideband users requesting channel access.

I. INTRODUCTION

Under a static frequency allotment scheme each licensed operator is allocated a frequency band in which its users can transmit. The licensed operator is charged with monitoring and regulating user transmissions. Fixed channel allotments allow one to minimize interference by users of adjacent frequency bands. With the addition of new services and the increase in the number of users requiring radio frequency access, underutilization of spectral resources is of concern. It has been demonstrated [1] that the spectrum may not be fully utilized either on a geographical or at a temporal level. These unused portions of the spectrum may offer opportunities for unlicensed user transmissions and applications. With the growth of the number of wireless users, efficient and secure usage of the radio spectrum has taken on greater importance.

Interference and other factors that negatively impact the quality of service for the licensed users remains an obstacle to simply overlaying secondary users onto the licensed spectrum. The introduction of cognitive radio is seen as one possible solution for managing multiple networks using the same radio spectrum. Cognitive radio refers to smart radio that has capability to detect and adopt appropriate transmission parameters to accommodate user’s communication needs[2][3]. Cognitive radios appear to be a prefect fit for managing the spectrum for co-operating networks. However it is uncertain to what extent such unlicensed user activity may impact the performance of the exiting licensed user pool. The design and development of cognitive radios have motivated researchers to investigate method to better utilize the radio spectrum.

Understanding the performance of systems that share a common spectral resource is essential for the design of communication networks that may use cognitive radio. Work has been done on modeling the performance of narrowband (NB) and wideband (WB) systems. Epstein and Schwartz [4] proposed an admission control algorithm. A multiclass prediction method is used for fair bandwidth sharing and blocking control. Kim[5] extended their work by creating a more general model which fairly admits NB and WB users. Gimpelson’s [6] access model for NB and WB sources allowed WB callers to be given an advantage when they can not gain access. In order to equalize the accessibility of both system Gimpelson introduced a switching model based on the blocking probabilities of both systems. Xing et al.[7] proposed a Markov model for the dynamic access allocation in open spectrum for NB and WB sources in which fairness based on airtime for both systems was analyzed. Giorgetti [8] investigated performance of UWB system in the presence of Bluetooth-like and GSM-like signals assuming Rake reception in Nakagami channels.

The steady state blocking probabilities have been used to model performance of many communications systems such as multi-hop wireless networks[9], cellular[10] and optical networks[11]. However, their objectives and system characteristics are different than presented in this paper. Blocking probabilities in most optical networks papers are modeled based on wavelength availability. Multi-hop networks transmit by hoping the signal through multiple nodes before the signal reaches its final destination. On the other hand, cellular networks communicate with the neighboring cells. We use a simple multi-server queueing Markov model to evaluate the impact of spectrum sharing between NB and WB systems.

There are numerous interesting applications, both commercial and military, to which this model applies. Some of these applications are Bluetooth and Ultra-Wideband (UWB). Both
technologies have different channel requirements, transmission rates and ranges. However their common characteristic is that they are both assigned to operate in unlicensed bands. Unlicensed bands are leaving a realm of opportunities for emerging wireless systems that have capability of co-operating in the same spectrum with other emerging applications. Such applications must be robust to interference and have low probability of detection. Weiss et al.

In this paper, we examine the performance of two types of traffic sources NB and WB which share a common spectrum. We assume that NB users are licensed users and WB users are unlicensed users. Both user types operate in a common spatial range. We investigate under which conditions WB users can transmit in the same spectrum without significantly interfering with the transmission of NB users. Performance measure of both systems is evaluated through the blocking probability. In order to control the performance of both sources population size of WB users may be limited.

Our approach focuses on improving the performance of coexisting networks while increasing the efficiency of spectrum usage. Our analysis considers spectral access based on channel availability. The controlling factors in the system performance can be gleaned from the degree of blocking or denial of access experienced by NB users [13]. To do so we will examine the blocking performance of NB and WB users requesting access to a fixed number of channels. If the users of each type is drawn from a infinite population we may reduce the blocking experienced for both types by increasing the number of the available channels. However, the blocking probability of WB users will always be greater than that experienced by NB users.

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This paper is organized as follows. In Section II we describe system model and define system parameters. System is analyzed with the general Markov model and blocking probabilities for both NB and WB systems are given in Section III. In Section IV we discuss the impact of coexistence of NB and WB sources in the same radio spectrum and we quantify the trade offs between the coexisting users. The spectral efficiency is also investigated in this section. Section V concludes the paper.

II. MODEL AND SYSTEM PARAMETERS

Consider the multichannel communication system shown in Fig. 1. In this model users are classified into two types- WB and NB users. The average time between channel requests of NB users is given by \( \lambda^{-1} \) and the average duration of the transmission is \( \mu^{-1} \). The average time between WB requests \( \lambda_n^{-1} \) where the subscript \( n \) represents the number of WB user occupying channels. The average time duration of WB transmission is \( \bar{\mu}^{-1} \). Utilization of NB and WB sources are defined respectively as \( \rho = \frac{\lambda}{\mu} \) and \( \rho_n = \frac{\lambda_n}{\bar{\mu}} \).

The total number of available channels is equal to \( K \). NB users are taken to use one channel for their transmissions. WB user requires \( M_W \) channels for its transmission. Both user types behave as independent Poisson processes and are combined into a single Markov process model. Users are given an equal priority and are served in the first come first served fashion. User requests are not queued, therefore, the user will be blocked if channels are not available for transmission at the time the user arrives.

Both sources are assumed to be drawn from a infinite population of users where WB users are allowed to request an allotment until the number of WB users reach \( N_{pop} \). Once the number of WB users in the common spectrum reaches \( N_{pop} \) the next arrival for WB user will not be scheduled until at least one of the WB users leaves the system.

A schematic representation of the channel occupancy is shown in Fig. 2. The state of the spectrum is described in terms of the number of users of each type occupying some subset of the \( K \) relegated radio channels. The maximum number of WB users that can transmit simultaneously is defined as \( N_W = \left\lfloor K/M_W \right\rfloor \). Since NB user requires only one channel for transmission maximum capacity of NB users in the spectrum is \( K \). However the number of WB users in the system will be limited to \( \min(N_{pop}, N_W) \). The number of NB and WB users of the spectrum are denoted by the indices \( j \) and \( i \) respectively. The probability of making a transition from one index pair to another is modeled by a two dimensional continuous time Markov chain. Each directed edge is labeled by probability of making a transition between connected nodes in time \( \Delta t \). In the steady state, the probability of having \( j \) NB and \( i \) WB users is given by \( p(i, j) \). The probability of all channels being idle is given by \( p(0, 0) \). Any available channel in the provided spectrum can be used upon arrival of a user. It is assumed once the channel is assigned to a user, the channel will remain occupied until user completes its transmission.

To make the transition between the idle state \((0, 0)\) to state \((0, 1)\) requires an arrival of NB user and occurs with probability \( \lambda \Delta t \). On the other hand, going from idle state \((0, 0)\) to state \((1, 0)\) requires the arrival of a single WB user which occurs with probability \( \lambda_0 \Delta t \). We model arrivals from a pool
Fig. 2. State transition diagram.

of $\hat{N}_{\text{pop}}$ WB users as follows

$$
\hat{\lambda}_n = \begin{cases} 
\beta(\hat{N}_{\text{pop}} - n) & 0 \leq n \leq \hat{N}_{\text{pop}} \\
0 & n > \hat{N}_{\text{pop}} 
\end{cases}
$$

(1)

Finite population model converges to infinite population model as $\hat{N}_{\text{pop}} \to \infty$. Substituting an expression $\hat{\lambda} = \hat{N}_{\text{pop}}\beta$ into Eqn(1) expression for WB arrivals for $0 \leq n \leq \hat{N}_{\text{pop}}$ may be represented as follows

$$
\hat{\lambda}_n = \hat{\lambda} - \frac{\hat{\lambda}}{\hat{N}_{\text{pop}}}
$$

(2)

with the limit as $\hat{N}_{\text{pop}}$ goes to infinity one can note that $\hat{\lambda}_n$ converges to $\hat{\lambda}$.

Transition rates corresponding to transition from higher to lower state correspond to average service rates $\hat{\mu}$ and $\mu$. When all $K$ channels are occupied, a NB user is blocked. WB source will be blocked when number of available channels is less than $M_W$.

### III. Blocking Probability for Wide and Narrow Band Users

Blocking of NB and WB users are described in terms of steady state probabilities and we defined blocking as a probability that system will be in any of the state for which given user will be blocked. The blocking of both user types will dependent on their offered loads. To improve the blocking performance of both user types one can limit the number of WB users that can simultaneously occupy the system. We construct a more general model that can be used to characterize general arrival distribution of WB users for this case. Statistical equilibrium state equation describing Markov chain given in Fig. 2 is

$$
p(i, j) [j\mu + i\hat{\mu} + \hat{\lambda}_i + \lambda] = p(i, j-1)\lambda + p(i-1, j)\hat{\lambda}_{i-1} + p(i, j+1)(j+1)\mu + p(i+1, j)(i+1)\hat{\mu}
$$

(3)

for $i = 0, \min(\hat{N}_{\text{pop}}, \hat{N}_W)$ and $j = 0, K - iM_W$. The probability is taken to equal zero when either or both indices are equal to zero. Since the number of users in the system is independent of each other, steady state probabilities $p(i, j)$ can be represented as product of two functions $f_i$ and $g_j$. The probability of $i$ WB users occupying the spectrum is given by function $f_i$. Whereas the probability of $j$ NB users occupying the spectrum is given as function $g_j$. The distribution functions satisfy the recursions

$$
g_j = g_{j-1}\frac{\rho_j}{j} \quad \text{Eqn} \quad (4)
$$

$$
f_i = f_{i-1}\frac{\hat{\rho}_i}{i} \quad \text{Eqn} \quad (5)
$$

Expanding the expressions we note that both expressions can be simplified to

$$
g_j = g_0 \frac{\rho_j}{j!} \quad \text{Eqn} \quad (6)
$$

$$
f_i = f_0 \frac{\hat{\rho}_i}{i!} \prod_{n=0}^{i-1} \hat{\rho}_n \quad \text{Eqn} \quad (7)
$$

Combining the expressions for $f_i$ and $g_j$ steady state probabilities are defined as

$$
p(i, j) = p(0, 0) \frac{\rho_j}{j!} \prod_{n=0}^{i-1} \hat{\rho}_n \quad \text{Eqn} \quad (8)
$$

where the probability of all channel being idle is $p(0, 0) = f_0 g_0$. 

342
Blocking of NB and WB users are given by \( \text{Prob}[NB] \) and \( \text{Prob}[WB] \) respectively

\[
\text{Prob}[NB] = \sum_{i=0}^{\min(\hat{N}_W, \hat{N}_{pop})} p(i, K - i \hat{M}_W) \tag{10}
\]

\[
\text{Prob}[WB] = \sum_{i=0}^{\min(\hat{N}_W - 1, \hat{N}_{pop} - 1)} \sum_{j=M_W - i}^{K - i \hat{M}_W} p(i, j) + \left[ \sum_{j=0}^{K - \hat{N}_W \hat{M}_W} p(\hat{N}_W, j) \right] \tag{11}
\]

where,

\[
\alpha = \begin{cases} 
0 & (\hat{N}_W + 1) > \hat{N}_{pop} \\
1 & \text{otherwise} 
\end{cases} \tag{12}
\]

The magnitude of bracketed term in Eqn(12) is governed by constant \( \alpha \). When \( \hat{N}_W + 1 > \hat{N}_{pop} \) the bracketed term equates to zero. We can note that reducing the number of WB users below this threshold will result in a lower blocking of WB users.

The computational complexity of this model increases with the number of available channels. We note that as the population size of WB users approaches \((\hat{N}_W + 1)\), the blocking of NB users also increases and it approaches the blocking of the infinite population case, \( \hat{N}_{pop} \).

When both NB and WB sources have the infinite population of users the blocking of WB users is always higher than the blocking for NB users work. The blocking for WB users is bounded by blocking for NB users

\[
\text{Prob}[WB] = \text{Prob}[NB] + \sum_{i=0}^{\hat{N}_W - 1} \sum_{j=M_W(\hat{N}_W - i - 1)}^{K - i \hat{M}_W} \frac{\hat{M}_W(\hat{N}_W - i - 1) \rho^j j!}{\prod_{n=0}^{i-1} \hat{N}_W - n + 1} \tag{13}
\]

Therefore, when both sources have infinite user population one may be able to reduce the blocking of WB but the WB users will never have a lower blocking probability than NB users. Therefore, as \( \hat{N}_{pop} \) approaches \( \hat{N}_W \) the result converges to infinite population blocking. In the Section IV we show that

\[
p(0, 0) = \frac{1}{\sum_{i=0}^{\hat{N}_W} \prod_{n=0}^{i-1} \rho_n \sum_{j=0}^{K - i \hat{M}_W} \frac{\rho^j j!}{j!}} \tag{9}
\]

Fig. 3. Simulation and Analytical Results, \( K = 12, \hat{M}_W = 2, \hat{N}_{pop} = 6, \rho_n = \{0.80, 0.67, 0.53, 0.40, 0.27, 0.13\}, \lambda_n = \{4.8, 4.0, 3.2, 2.4, 1.6, 0.8\}, \beta = 0.8 \) and \( \tilde{\mu} = 6 \).

IB users are less affected as the population size of WB users is lowered.

IV. RESULTS

In this section we demonstrate the influence of system parameters on the degree of blocking for both user types. WB and NB blocking are demonstrated for the case when the average arrival rate for NB users is fixed to \( \lambda = 3 \). All of the tabulated results are shown as function of \( \rho \).

Increasing the utilizations of WB and NB users will also increase their blocking probability. Blocking of WB and NB users are shown in Fig. 3. As \( \rho \) increases blocking probabilities in general increase as well. This figure demonstrates that analytical results are supported by simulation results. The average arrival parameter for WB users was taken to be \( \beta = 0.8 \). The population size of WB users for this case is limited to \( \hat{N}_{pop} = 6 \) and the average service rate for WB users was taken to be \( \tilde{\mu} = 6 \). We assume that number of available channels in the spectrum is \( K = 12 \) and that WB user requires \( \hat{M}_W = 2 \) channels to transmit. Differences between simulation and analytical results are due to number of realizations of WB arrivals in the simulation which was taken to equal \( 10^6 \).

Blocking performances of WB and NB users may be improved relative to WB arrivals. Limiting population size of
WB users improves the performance of both user types. Fig. 4 demonstrates the improvement in blocking for NB and WB users as the population of WB users is limited from infinite to finite population. Blocking for both user types is shown for $N_{\text{pop}} = \infty$ and $N_{\text{pop}} = 4$. For both cases the number of available channels in the spectrum is set to $K = 12$, $M_W = 2$ and $\hat{\mu} = 6$. For $N_{\text{pop}} = \infty$ average arrival parameter $\rho$ is set to $\lambda = 4.2$. The corresponding utilization is $\hat{\rho}_n = 0.8$ for all $n$. In order to be able to compare infinite and finite models we initialize both $\hat{\rho}_0$ to the same value, $\hat{\rho}_0 = 0.8$. Therefore, $N_{\text{pop}} = 4$ and average arrival parameter $\beta = 1.2$. For the lower values of $\rho$ one can see that blocking of both user types is lower when the population of WB users is limited. Also, blocking of WB users is lower for lower values of $\rho$. As $\rho$ increases blocking of both user types increases. Furthermore, WB blocking for limited population case becomes higher than the blocking of NB users as $\rho$ increases.

Decreasing the population size of WB users blocking of both user types will also significantly decrease. Fig. 5 compares the blocking for NB users for two different population sizes of WB users. This figure demonstrates the blocking for $N_{\text{pop}} = 2$ and $N_{\text{pop}} = 5$. We assume that number of available channels for both cases is $K = 12$ and that WB users require $M_W = 2$ channels to transmit data. Parameters for WB source are set as follows $\hat{\mu} = 6$ and $\beta = 0.8$. It can be seen that by increasing the value of $N_{\text{pop}}$ the blocking of NB users will also increase. Increasing blocking of NB users occurs due to the transmission of more WB users in the system. When $N_{\text{pop}} = 2$ only 4 channels may be occupied by WB users simultaneously, and in second case for $N_{\text{pop}} = 5$, 10 channels may be occupied at the same time by WB users. Similar dependence on arrival rates and population size of WB users can be seen in WB blocking. Fig. 6 compares the WB blocking for the same set of parameters as used in Fig. 5. Blocking of WB users decreases with decreasing $N_{\text{pop}}$. It is shown that by properly controlling arrival rates and population size of WB users we may significantly decrease the blocking of both user types.

By increasing the number of channels in the spectrum blocking of both user types decreases. We define number of excess channels as $n_e = K - N_{W}M_{W}$. Excess channels are provided only for NB users, and therefore $n_e < M_{W}$. Fig. 7 demonstrates the impact of providing excess channels on performance of NB users. For this case we assume $M_W = 3$, $N_{\text{pop}} = 5$, $\beta = 0.8$ and $\hat{\mu} = 6$. Number of channels in the spectrum are increased from $K = 18$ to $K = 20$. When $K = 18$ there are no excess channels in the system, $n_e = 0$. We compare this case to $K = 19$ when $n_e = 1$ and $K = 20$ with $n_e = 2$. Providing additional channels for NB users improves their performance by reducing the blocking. Reducing the blocking for NB users correspondingly reduces the blocking for WB users. Fig. 8 shows the blocking of WB users as number of excess channels is increased from $n_e = 0$ to $n_e = 2$. Other system parameters are the same as in Fig. 7. In both cases, providing excess channels to the spectrum decreases the blocking for users of both sources. As the number of excess channels increases the performance of both system improves.

We have demonstrated that by limiting and decreasing the population of WB users we may improve the performance of
Fig. 6. Blocking Probability for WB Users, $K = 12, M_W = 2$, Case 1: $N_{pop} = 2, \hat{\rho}_n = \{0.27, 0.13\}$, Case 2: $N_{pop} = 5, \hat{\rho}_n = 0.67, 0.53, 0.4, 0.27, 0.13$.

Fig. 7. Blocking Probability for NB Users as Number of Excess Channels Increases, $M_W = 3, N_{pop} = 5, \hat{\rho}_n = \{0.67, 0.53, 0.4, 0.27, 0.13\}$.

Fig. 8. Blocking Probability for WB Users as Number of Excess Channels Increases, $M_W = 3, N_{pop} = 5, \hat{\rho}_n = \{0.67, 0.53, 0.4, 0.27, 0.13\}$.

NB and WB users. This was done to allow spectrum sharing between two different user types in order to increase the utilization of spectrum. We analyze the probability of system being idle in order to evaluate the level of spectrum usage. Fig. 9 shows that the probability of system being in the idle state decreases as $\rho$ increases. This corresponds to increasing blocking probabilities of both sources. Fig. 9 depicts 3 cases of the system being in the idle state: (1) $N_{pop} = \infty$ and $\hat{\lambda} = 4$, (2) $N_{pop} = 5$, and (3) $N_{pop} = 2$. System parameters are set to be $K = 12, M_W = 2, \beta = 0.8$ and $\mu = 6$. System spends less time in idle state as the number of WB users in the system increases. Also, one can see that probability of system being in the idle state is lower when $N_{pop} = \infty$ than when $N_{pop}$ is limited. This corresponds to the blocking probabilities in which blocking for WB and NB users is higher at constant $\hat{\rho}_n$.

V. CONCLUSIONS

Performance model for blocking for WB and NB users sharing the common radio spectrum was presented. Assuming that NB users are licensed and WB users are unlicensed users in the spectrum, it was shown that by controlling average arrival rates and the population size of unlicensed users it is possible to transmit both types of traffic using the same radio spectrum without adversely disrupting the quality of
Blocking Probability $ρ$

$N_{\text{pop}} = \infty$

$N_{\text{pop}} = 5$

$N_{\text{pop}} = 2$

Fig. 9. Probability of System Being in Idle State, $K = 1.2$, $M_W = 2$,
Case 1: $N_{\text{pop}} = \infty$ and $ρ_{n} = 0.67$, Case 2: $N_{\text{pop}} = 5$ and $ρ_{n} = \{0.67, 0.53, 0.4, 0.267, 0.13\}$, and Case 3: $N_{\text{pop}} = 2$, $ρ_{n} = \{0.27, 0.13\}$

service for licensed users. As the population size of WB users increases the blocking of both NB and WB users increases as well. In order to enhance the performance of NB users and allow more WB users to transmit simultaneously, we decreased the offered load of WB users as its user population size in the spectrum increased. By discouraging the average arrival rates of WB users we were able to decrease blocking of both sources. Providing excess channels in the spectrum reduced the blocking of NB users. Reducing the blocking of NB users correspondingly reduced the blocking of WB users.

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REFERENCES
