

Channel Assignment for Time-Varying Demand

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Abstract— This paper presents an integer programming model for dynamic channel assignment (DCA) under space and time-varying traffic demand. The algorithm minimizes the number of channels required to satisfy the traffic demand using a threshold based decision criterion on the carrier-to-interference ratio. A neighborhood based search procedure uses the most recent channel state information to perform a feasible assignment when the demand changes. This technique accelerates the convergence of the algorithm to a local minimum and allows an evaluation of channel gains obtained with increasing neighborhood sizes. This procedure will also minimize the number of channel reassignments in cells whose demand is time-invariant. The performance of a greedy sequential channel assignment heuristic(SA) is examined relative to the spatial distribution of the cells with time-varying demand. Channel gains obtained with DCA relative to the SA scheme range from 30 – 40% for the examples discussed.

I. INTRODUCTION

Wireless cellular networks are bandwidth and power limited. A finite frequency spectrum is available for provision of commercial services. Based on the service requirements, the allocated spectrum is divided into a number of channels. Channels are assigned to geographical regions based on expected traffic demand. The reuse of frequencies at distances large enough to minimize co-channel interference is a basic design principle in cellular networks. Most existing networks have a fixed number of channels assigned (FCA) permanently to each cell for its exclusive use. This arrangement is inefficient for wireless transmission of packet voice, video and data services. The traffic patterns that characterize packet data and video have been found to exhibit high temporal variability [1], [2]. In particular, such packet traffic exhibits persistence in both underload and overload conditions for durations longer than that predicted by classical negative exponential distributions. This state-of-affairs will lead to both under utilization of resources and higher blocking with FCA schemes for bursty traffic sources. The requirement of larger channel bandwidths for broadband transmission will also impose a stronger requirement on optimal allocation of channel resources. Dynamic channel assignment (DCA) schemes that can perform at the time-scale of traffic variation are there-

fore an important component in future wireless networks. Channel assignment schemes that minimize the overhead involved in reassignment of existing calls, while maximizing channel utilization are of particular interest.

A survey of fixed, dynamic and hybrid channel assignment schemes is provided by Katzela and Naghshineh [3]. The simplest modifications to FCA are based on borrowing channels from the richest neighboring cells to minimize future call blocking probability. Anderson [4] discusses simulation studies of these algorithms and shows that the number of search steps required can limit the performance of the approach. Modifications to reduce the number of search steps have also been considered [5]. This involves channel ordering schemes where the fixed-to-borrowable channel ratio is dynamically varied according to changing traffic conditions.

In DCA there is no fixed relationship between channels and cells and all channels are available for assignment to all cells. Channel assignment takes place through minimization of a cost function such as the allocated bandwidth under the constraint that channel reuse takes place above specified interference levels. Many instances of this problem are computationally difficult. Murphey et al. [6] provide a comprehensive survey of algorithmic approaches to the problem. Techniques may be broadly classified as graph theoretic, mathematical programming approaches and meta-heuristic search, or some combination thereof. In a typical graph-theoretic abstraction each transmitter is represented by a graph node and two transmitters share a graph edge if using the same channel could create interference. Graph coloring and Integer programming (IP) formulations have been used here [7]. IP formulations also exist for some non-graph-theoretic abstractions. Meta-heuristics of various types have also been employed. Capone and Trubian [8] use a tabu search meta-heuristic that leverages search history data to avoid unproductive parts of the search space. Smith and Palaniswami[9] use a combination of neural-networks, simulated annealing and steepest descent heuristics to solve a nonlinear IP formulation of the problem.

Although a variety of algorithmic models and solution techniques have been proposed for DCA, the performance of such algorithms in the context of varying traffic demand has been examined to a lesser extent. Argyropoulos et al.

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[10] showed that in the presence of spatial load-imbalance DCA produced much greater improvements in the performance than FCA relative to gains for uniform load distribution. Everitt and Mansfield [11] assumed that the mean traffic in each cell is Gaussian distributed and showed that DCA is more resilient to traffic volatility than FCA.

This paper studies dynamic channel assignment using a time-step simulation with temporal demand variation. A traffic demand model and channel assignment model are invoked for each time step. The traffic demand model calculates the number of calls that must be satisfied for each transmitter. The channel assignment model assigns channels to transmitters while satisfying three types of constraints and seeking to optimize the assignment in two ways. The constraints are: 1) satisfy demand for each transmitter, 2) assign the same channel to two transmitters only if there is enough geographic distance between the transmitters, and 3) limit the computation time. The optimization criteria are: 1) minimize the number of channels used, and 2) minimize changes in channel assignments across time steps.

Section II discusses the demand models. Section III presents the channel assignment model. This solves a sequence of IP models. Each IP model's formulation allows more reassignments with respect to an assignment from the previous time step. Assignment changes are controlled using a geographic neighborhood policy. Each model is solved with an objective of minimizing the number of channels used, subject to constraints on demand satisfaction, frequency-distance interference, and running time, plus constraints that implement the neighborhood policy. Section IV presents results of dynamic channel assignment. Section V concludes the paper.

II. TRAFFIC DEMAND MODEL

Each cell is assumed to generate either a constant demand rate (Type-C) of one channel per unit time or a variable demand rate (Type-V) where the demand can be greater than one. The channel holding time is assumed to be one time unit. Overload conditions occur when one or more sources are of type V . For packet data, a limited queueing space is often available to facilitate channel search, allocation and setup procedures. A type V cell that persists in the overload state for a finite time therefore demands an increasing number of channels in each successive time step. This is the traffic scenario considered in this study.

The demand generated by each cell is independent of the traffic in other cells. Variable demand is modeled using a two-state discrete time Markov chain. This traffic model is easily extended to larger number of states. Markov chains afford a tractable model for controlling the level of temporal correlation in the traffic. The correlation measure used in this work is the expected time a source remains in the

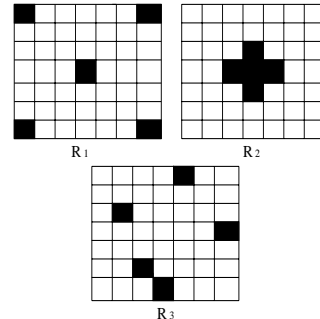


Fig. 1. Spatial distribution of variable demand cells (shaded).

overload state. If the probability of transition from type V to C demand is β , the expected duration in the overload state is $T_{OL} = \beta^{-1}$. Dynamic channel allocation will be examined for an increasing range of $T_{OL} : 1, 2, \dots, 10$.

The minimum channel requirement is also highly dependent on the spatial distribution of the type V sources. To demonstrate this effect three representative spatial distributions R_1 , R_2 and R_3 are overlaid on a 7×7 grid of square cells as shown in Fig. 1. The channel requirements with time are a function of the number of type V cells that exist within the frequency reuse distance. R_1 and R_2 represent extreme cases of deterministic spatial configuration where the distance between type V cells is maximum and minimum respectively, relative to the grid size. R_3 is an example of a randomly distributed configuration. The performance of the channel assignment model described in Section 3 will be examined for these three cases subject to increasing T_{OL} . First as a baseline comparison for resource utilization the number of channels that may be allocated using FCA is estimated using an effective bandwidth calculation for the Markov sources.

A. Effective Bandwidth Based Assignment

The capacity requirements of independent Markov sources multiplexed on a common set of resources can be estimated using an effective bandwidth approximation. Let Q_m denote the infinitesimal Markov generator of m identical multiplexed sources. This can be determined from the $m - fold$ Kronecker sum of the single source generator Q . For two state sources, the dimension of Q_m is $m + 1$. Let R_m denote the diagonal source rate matrix with elements r_i , $i = 0, 2, \dots, m$ representing the state dependent channel demand. The number of channels required to service this demand can be estimated for a given bound on the probability of number in the queue n_q . It is assumed that the asymptotic decay rate of the complementary distribution $Pr(n_q > buffer) = P_{buffer} \approx e^{-z n_q}$. The capacity required for a specified buffer size and probability P_{buffer} may be derived as the maximal real eigenvalue of the ma-

TABLE I
CHANNEL REQUIREMENTS UNDER FCA

Effective Bandwidth Based Channel Allocation			
T_{OL}	R_1	R_2	R_3
2	11	19	15
3	12	24	18
4	13	29	21
5	14	34	24

trix $\left[\frac{Q_m}{z} + R_m\right]$. The single source modeled by a two state Markov chain is assigned demand rates $r_1 = 1$ and $r_2 = 2$. The channels required for $m = 1, 2, 3$ multiplexed sources that satisfy performance constraints $buffer = 1$, $P_{buffer} = 10^{-3}$ were computed. These values are represented by $S_V(m)$ and provide an estimate of channel requirement of m type V sources located within a channel reuse distance. This is referred to as a type V cluster of size m . The total number of channels required for the three spatial configurations in Fig. 1 can be estimated as $S(T_{OL}) = S_C + \sum_{m=1}^3 N_{C_V}(m) * S_V(m)$, where $S_C = 9$ is fixed as the channel requirement for type C cells for a reuse distance of two, $N_{C_V}(m)$ represents the number of type V clusters of size m that reside within the specified reuse distance. The reuse distance of greater than two cells considered here is consistent with the co-channel interference threshold used in the IP model. The results are given in Table I and show that the incremental channel requirement for each T_{OL} is proportional to the cluster size of variable demand cells. The efficiency of DCA will be evaluated relative to these FCA assignments.

III. CHANNEL ASSIGNMENT MODEL

The traffic demand for a given simulation time step t is used to formulate the channel assignment problem for time t as a sequence of IP models. At the core of each IP model in the sequence is a common channel assignment formulation that seeks the minimum number of channels satisfying the traffic demand, a set of frequency-distance interference constraints, and a computation time limit. We refer to this as a minimum-order channel assignment IP formulation for static demand. This formulation is augmented by a set of constraints that varies across the sequence of IP models. This set controls whether or not the channel assignments for a transmitter at time t are allowed to change from the best channel assignments found for that transmitter at time $t - 1$. A geographic neighborhood policy specifies constraints for each model in the sequence that allow more reassignments to change than for the previous model in the sequence. The geographic neighborhood approach and the specifics of the static demand IP formulation are

described next. Following this is the approach used to add constraints to the static demand formulation for a given time step to implement the neighborhood policy.

A. Geographic Neighborhood Approach

Each transmitter is associated with a cell in an r -row square grid. For each $cell(i, j)$ in the grid, where i denotes row and j denotes column, a geographic neighborhood of size h is defined: $N_h(i, j) = \{cell(k, l) : |i - k| \leq h, |j - l| \leq h\}$. $N_h(i, j)$ is used to formulate an IP model. $IP_h(t)$ based on a neighborhood of size h at time t . Neighborhood size limits reassignments as follows: let $D(i, j, t)$ denote the demand for $cell(i, j)$ at time t , calculated as in Section II. For a given value of h , $D(i, j, t)$ is compared with $D(i, j, t - 1)$. If the demand has changed, the assignments for cells in $N_h(i, j)$ are allowed to change at time t . Cells that are not in a neighborhood of any cell whose demand changes from time $t - 1$ to time t keep their time $t - 1$ assignments at time t . The core of $IP_h(t)$ is described below in Section III(B). That section also shows how to add constraints to a core model to obtain $IP_h(t)$. For each t , a sequence of IP models for increasing neighborhood sizes is created and solved: $IP_0(t), IP_1(t), \dots, IP_{r-1}(t)$.

B. IP Model

The minimum order frequency assignment IP formulation for static demand has two groups of binary decision variables: 1) variables that show if a given transmitter is assigned to a given frequency, and 2) variables that reflect whether or not a given frequency is used in the solution. The objective function of the canonical formulation minimizes the sum of the type (2) variables. Type (2) variables are linked to type (1) variables via a set of constraints. In the description below, the square cell grid has r rows and c columns. There are q available frequencies (i.e. channels). B is the C/I ratio threshold value where C is signal to be acquired and I represents the sum of interfering signals. Its magnitude determines the reuse distance. $d(i, j, k, l) =$ Euclidean distance from the center of $cell(i, j)$ to the center of $cell(k, l)$. $S(i, j, k, l) =$ strength of the signal received due to transmission between $cell(i, j)$ and $cell(k, l)$. The co-channel interference between $cell(i, j)$ and $cell(k, l)$ is calculated as:

$$S(i, j, k, l) = \frac{1}{d(i, j, k, l)^\alpha} \quad (1)$$

where α is a path-loss exponent. The integer binary decision variables are:

$$A(i, j, f) = \begin{cases} 1 & \text{if frequency } f \text{ assigned } cell(i, j) \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

$$a(f) = \begin{cases} 1 & \text{if } \sum_{i,j} A(i, j, f) \geq 1 \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

The model can be stated as follows:

$$\min \sum_{f=1}^q a(f) \quad (4)$$

subject to:

$$(rc)a(f) - \sum_{i,j} A(i,j,f) \geq 0$$

$$\text{for } 1 \leq f \leq q, 1 \leq i \leq r, 1 \leq j \leq c \quad (5)$$

$$\sum_{f=1}^q A(i,j,f) \geq D(i,j,t) \quad \text{for } 1 \leq i \leq r, 1 \leq j \leq c \quad (6)$$

$$\left[\frac{S(i,j,i,j)}{B} - A(i,j,f) \right] \sum_{(k,l) \neq (i,j)} S(i,j,k,l) -$$

$$\left[\frac{S(i,j,i,j)}{B} A(i,j,f) \right] \geq \sum_{(k,l) \neq (i,j)} S(i,j,k,l) A(k,l,f)$$

$$\text{for } 1 \leq f \leq q, 1 \leq i, k \leq r, 1 \leq j, l \leq c \quad (7)$$

Eq. (5) links the binary variables $A(i,j,f)$ that show if a given transmitter is assigned to a given frequency to the binary variables $a(f)$ that reflect whether or not a given frequency is used in the solution. Eq. (6) is the set of demand constraints. Eq. (7) contains frequency-distance interference constraints that allow two transmitters to be assigned the same channel only if there is sufficient distance between the transmitters. These constraints are a linearization of a set of nonlinear constraints. Channel reuse for the nonlinear constraints is based on the C/I ratio for a given frequency.

To provide a reasonable initial upper bound on the number of channels, we use a greedy sequential assignment (SA) algorithm. The SA is based on a sequential and fair assignment strategy that uses the same frequency-distance interference conditions as the IP model. The SA scheme traverses the cells in a sequential order assigning one channel to each cell subject to the interference threshold constraint. This round-robin allocation is repeated until all the cells have the required demand.

As discussed in Section III(A), for time t the solution is obtained for a sequence of IP models of increasing neighborhood size. For a neighborhood of size h , cells that are not in an h -sized neighborhood of any cell whose demand changes from time $t-1$ to time t keep their time $t-1$ assignments at time t . If $cell(i,j)$ will keep its time $t-1$ assignment, an additional constraint of the form: $A(i,j,f) = v$, is included in the model, where v is the value of the time $t-1$ assignment. For each time t and an initial upper bound on the number of available channels, each IP model in the sequence is solved using the Mixed Integer Programming solver component of

the *CPLEX7.0*^{TM 1} optimization software package. An upper bound on CPLEX computation time is imposed. Note that if no upper bound were imposed, we would expect the minimum number of channels found for models in the sequence to satisfy: $IP_0(t) \geq IP_1(t) \geq \dots \geq IP_{r-1}(t)$ for each t . However, this may be violated when limiting the computation time.

The final assignment for time t is the assignment for the “best” neighborhood size for time t , where “best” corresponds to the size whose model solution yields the smallest minimum number of channels. If more than one neighborhood size produces this number of channels, the smallest of these neighborhood sizes is selected, as this minimizes the number of reassignments from time $t-1$ to t .

IV. RESULTS

The IP model was solved for the three spatial configurations of type V cells shown in Fig.1. In cases where the number of channels corresponding to the best integer solution remained constant over a large number of nodes in the Branch and Bound search, the search was terminated early. In these cases the solution is not necessarily optimal so the neighborhood constraint on the solutions is not necessarily enforced. For each spatial configuration, the demand generated by the marked cells increased at a uniform rate in time. The number of channels required at this sustained demand rate was determined in each time step using the neighborhood based restriction discussed above. The channel requirement $IP_n(t)$ was obtained for $t = 1, 2, \dots, 10$ for $n = 0, 1, 2$.

Figs. 2(a-c) depict the solutions obtained. For each case, the upper-bound solution of the sequential assignment SA algorithm is compared with the best IP solution across neighborhoods and that obtained with neighborhood size $n = 0$ denoted IP_0 .

The demand service rate was set equal to one per unit time interval. The cutoff time for CPLEX MIP solver was set to 900 seconds. The path-loss exponent $\alpha = 3.5$ and the C/I threshold value $B = 27234$, results in reuse distances greater than or equal to two cells. Type V cells generate demand of two channels each time unit, whereas type C cells generate demand of one each time unit. The rate of increase in number of channels required is a function of the average size of the type C neighborhood around a type V cell and the reuse distance allowed by the C/I constraint. For type C neighborhoods greater than the reuse distance, the increase in required channel capacity is simply the excess arrival rate $\lambda = 1$ in one type V cell. This is one channel for the example in Fig. 2(a). All three solutions exhibit this expected trend. The IP result is seen to converge to the local neighborhood based solution for large t . In case R_2 a variable demand cluster size of size 3 exists

¹CPLEX 7.0 is a trademark of ILOG Corporation

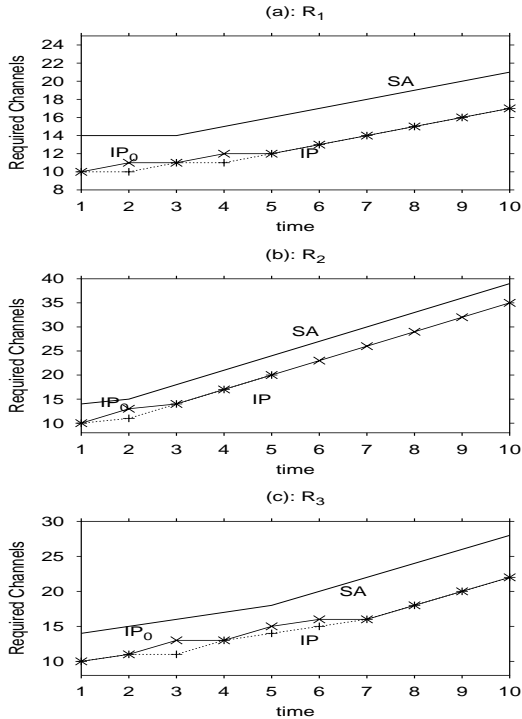


Fig. 2. Channel requirements using DCA

within the reuse distance. Therefore the solutions follow the expected increase of 3 channels each time unit. The configuration in R_3 having a cluster size of 2 also exhibits the expected channel requirement rate of 2 channels per time unit.

In comparison to SA schemes, IP models are seen to provide channel gains ranging from 30 – 40%. Relative to FCA assignments using effective bandwidth estimates, the channel gains range from 15 – 40%; the larger gains result for spatial configurations with separated variable demand clusters. The presence of two distinct trends for small and large t occurs due to the enforcement of the frequency reuse constraints. The slower rate of increase in channel requirements for small time results from the existence of assigned frequencies in type C cells that are reused in type V cells when their demand increases. There is an upper limit to the number of such available frequencies, that depends on the grid size. When the demand exceeds this reuse availability for large time, new channels demanded are assigned exclusively in the type V cells. This is also the reason why the IP and IP_0 solutions exhibit differences in the small time regime. Another reason can arise from the limit set on CPLEX computation time. The reassignment of existing frequencies is performed only when the neighborhood size is increased beyond zero.

In any dynamic assignment scheme a significant overhead results when existing assignments have to be inter-changed

to enable optimal assignments. The improvement achieved relative to a localized neighborhood constraint must be evaluated to determine if the maximum packing algorithms are warranted. In the examples considered, the efficiency of maximal packing is seen to be improved by less than 10%. This feature is however a function of the spatial distribution of the variable demand sources.

V. CONCLUSIONS

The computational difficulty of channel assignment for large problems makes it difficult to obtain optimal results. Based on this, one might intuitively expect minimizing the number of reassignments and minimizing the number of channels to be competing goals requiring a tradeoff. While this might actually be true for optimal solutions, our work suggests that exploiting locality of demand changes can allow these goals to be synergistic for dynamic channel assignment. The dynamic channel assignment algorithm proposed shows that the achievable channel gains depend on the spatial configuration of the cells with time-varying demands. In particular, the solutions can be reasonably well predicted using geometric descriptions of the spatial distribution. The results suggest that the monitoring and measurement of traffic descriptors based on space and time locality can increase the efficiency of implementation of mathematical programming algorithms.

REFERENCES

- [1] W. Willinger, M.S. Taqqu, R. Sherman, and D.V. Wilson, "Self-similarity through high-variability: Statistical analysis of ethernet lan traffic at the source level," *Proc. SIGCOMM'95*, p. 100:113, 1995.
- [2] V. Paxson and S. Floyd, "Wide-area traffic: The failure of poisson modeling," *IEEE/ACM Trans. Networking*, vol. 3, no. 3, pp. 226:244, 1995.
- [3] I. Katzela and M. Naghshineh, "Channel assignment schemes for cellular mobile telecommunication systems: A comprehensive survey," *IEEE Personal Commun.*, vol. 3, pp. 10:31, 1996.
- [4] L.G. Anderson, "A simulation study of dynamic channel assignment in a high capacity mobile telecommunication system," *IEEE Trans. on VT*, vol. 22, no. 4, pp. 210–217, 1973.
- [5] S.M. Elnoubi, "A new frequency channel assignment algorithm in high capacity mobile communication system," *IEEE Trans. on VT*, vol. 31, no. 3, pp. 125–131, 1982.
- [6] R. Murphey, P. M. Pardalos, and M. Resende, "Frequency assignment problems," *Handbook of Combinatorial Optimization*, vol. Supplement vol. A, pp. 295:377, 2000.
- [7] W.K. Hale, "Frequency assignment: Theory and applications," *Proc. of IEEE*, vol. 68, no. 12, pp. 1497:1514, 1980.
- [8] A. Capone and M. Trubian, "Channel assignment problem in cellular systems: A new model and a tabu search algorithm," *IEEE Trans. on VT*, vol. 48, no. 4, pp. 1252:1260, 1999.
- [9] K. Smith and M. Palaniswami, "Static and dynamic channel assignment using neural networks," *IEEE Journal Select. Areas of Commun.*, vol. 15, no. 12, pp. 238:249, 1997.
- [10] Y. Argyropoulos, S. Jordan, and S.P.R. Kumar, "Dynamic channel allocation in interference-limited cellular systems with uneven traffic distribution," *IEEE Trans. on VT*, vol. 48, no. 1, pp. 224:232, 1999.
- [11] D.E. Everitt and D. Mansfield, "Performance analysis of cellular mobile communication systems with dynamic channel assignment," *IEEE Journal Select. Areas of Commun.*, vol. 26, no. 8, pp. 125:139, 1982.