Alternating Current (AC) waveforms may be characterized with:

- Peak value ; Peak-to-Peak value ; Average value
- **Effective** or **Root Mean Square (RMS)** values

RMS value of $i(t) = I_m \cos(\omega t + \phi_i)$

$$I_{rms} = \sqrt{\frac{1}{T} \int_0^T I_m^2 \cos^2(\omega t + \phi_i) dt}$$

$$= \sqrt{\frac{I_m^2}{T} \int_0^T \frac{1}{2} (1 + \cos(2\omega t + 2\phi_i)) dt}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$
• The effective value provides a comparison between AC and DC circuits

• Average power absorbed by a resistance $R$ Ohms:

$$P = \frac{I_m^2}{2} R = \left[\frac{I_m}{\sqrt{2}}\right]^2 R = I_{rms}^2 R \text{ Watts}$$

• A DC source of $I_{rms}$ Amperes will supply same average power as an AC source of peak amplitude $I_m$ and effective value $I_{rms} = \frac{I_m}{\sqrt{2}}$

Note:

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$P_{avg} = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$
• Oscilloscope may be used to measure the peak-to-peak or peak values.

• The multimeter measures the effective or rms value of the signal.

• Power company specification of 110/220 Volts (available from outlet) is an RMS value.

Examples

1. Find effective value of current shown:

![Figure 1: Sawtooth function](image-url)
\[ I_{rms}^2 = \frac{1}{T} \int_0^T i^2(t) dt \]
\[ = \frac{1}{T} \int_0^T \frac{I_m^2 t^2}{T^2} dt \]
\[ = \frac{I_m^2}{3} \]
\[ I_{rms} = \frac{I_m}{\sqrt{3}} \]

2. Find effective value of \( i(t) = 4\cos(100t) + 5\cos(100t + 150^\circ) \)
\[ I = 4 \angle 0 + 5 \angle 150^\circ = 4 + 5\cos(150) + j5 \sin(150) \]
\[ = -0.33 + j2.5 = 2.52 \angle -83.157^\circ \]
\[ I_{rms} = 2.52/\sqrt{2} = 1.782 \text{ Amps.} \]
Apparent Power and Power Factor

- Average Power: \( P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) \)
- \( P = V_{rms} I_{rms} \cos(\theta_v - \theta_i) = S \cos(\theta_v - \theta_i) \)
- \( S = V_{rms} I_{rms} \) is termed Apparent Power (volt-amps: VA)
- The term \( \cos(\theta_v - \theta_i) \) is the power factor (pf)
- \( P = S \cdot pf \), \( pf = P/S = \cos(\theta_v - \theta_i) \)
- Angle \( \theta_v - \theta_i \) is the power factor angle
- Note: Impedance \( Z = \frac{V}{I} = \frac{V_m}{I_m} \angle \theta_v - \theta_i = \frac{V_{rms}}{I_{rms}} \angle \theta_v - \theta_i \)
- Power factor \( pf \): Cosine of angle of load impedance \( 0 < pf < 1 \)
- Load Resistive: \( Z = R : pf=1 \); Reactive: \( Z = jX : pf=0 \)
- In general \( Z = R + jX : pf \) is leading or lagging:
  - Leading power factor: Current Leads Voltage (Capacitive)
  - Lagging power factor: Current Lags Voltage (Inductive)
Power Factor Impact

• Power Factor has an economic impact on consumers of large power (industrial loads)
• A load with a low power factor that consumes $P$ watts of power draws higher current from a constant voltage source.
• Higher currents increase line losses and increase the amount of supplied power
• Such loads can be charged at a higher rate by power companies
• Ideally a pf of 1.0 is desired
• Most loads that consume a large amount of power are inductive
• Inductive load can be changed by adding capacitors to increase the pf towards unity value, thus optimizing cost.
Example

(a) Find apparent and real power supplied by the source.

Real Power Supplied: $V_{rms} I_{rms} \text{pf}$

Apparent Power: $S = V_{rms} I_{rms}$

Source Current $I_s = \frac{120\angle 0}{Z_{in}}$

$$Z_{in} = 1 + \frac{10(5 + j(2 \times \pi \times 60 \times 10 \times 10^{-3}))}{15 + j(2 \times \pi \times 60 \times 10 \times 10^{-3})}$$

$$= 1 + 4.05\angle 22.89 = 4.73 + j1.575 = 4.985\angle 18.42$$

Therefore:
- \( I_{s(rms)} = \frac{120}{4.985\angle18.42} = 24.07\angle-18.42 \)
- Apparent Power: \( S = 120 \times 24.07 = 2888.6 \text{ VA (2.888 kVA)} \)
- Power Factor: \( pf = \cos(0 + 18.47) = 0.9485 \) Lagging

(b) If a pf of 0.92 lagging is desired, to what value should the inductance be changed to?

\[
Z_{in} = A\angle\theta_v - \theta_i = A\angle\cos^{-1}(pf) \\
Z_{in} = R + jX \\
\angle\cos^{-1}(pf) = \tan^{-1}(X/R) \\
tan[\cos^{-1}(pf)] = X/R
\]

In circuit above, replace the inductor by unknown \( X \),

\[
Z_{in} = 1 + \frac{10(5 + jX)}{15 + jX}
\]
\[\frac{975 + 11X^2}{226} + \frac{j100X}{226}\]

\[\tan[\cos^{-1}(pf)] = \frac{100X}{975 + 11X^2}\]

\[\tan[\cos^{-1}(0.92)] = 0.426\]

\[0.426 = \frac{100X}{975 + 11X^2}\]

\[415.35 + 4.686X^2 - 100X = 0\]

Solving for X:

\[X = 5.649 \text{ or } X = 15.691\]

Using \(\omega L = X\) and \(\omega = 120\pi\)

\(L = 14.9mH \text{ or } L = 41.62 \text{ mH}\)
Complex Power

- Average \((P)\) and Apparent power \((S)\) are both real quantities
- Consider power represented as a complex number
  \[ S = \frac{VI^*}{2} = \frac{V_m I_m}{2} \angle \theta_v - \theta_i = V_{rms} I_{rms}^* \]
- Magnitude: \( |S| = \frac{V_m I_m}{2} \): Apparent Power
- \( S = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + j \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) \)
- Real part of \( S \) is \( \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) \) is Average Power \( P \)
- \( S = P + jQ \)
- Where \( Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) \) is Reactive Power
• Units
  – Average Power $P$ : Watts
  – Complex Power $S$ : volt-amps (VA)
  – Reactive power $Q$ : volt-amps reactive (VAR)

• In terms of load impedance $Z$

\[
Z = \frac{V}{I} = \frac{V_{rms}}{I_{rms}}
\]

\[
S = V_{rms}I^*_rms
= Z I_{rms}I^*_rms
= I^2_{rms}Z = \frac{V^2_{rms}}{Z}
\]
Since $Z = R + jX$

\[
S = I_{\text{rms}}^2 Z \\
= I_{\text{rms}}^2 [R + jX] = P + jQ
\]

Matching Real and Imaginary Parts

\[
\begin{align*}
P &= Re[S] = I_{\text{rms}}^2 R \\
Q &= Im[S] = I_{\text{rms}}^2 X
\end{align*}
\]

- Complex power incorporates all power information of a load
- Real power $P$ is the average power in watts delivered to the load
- Reactive power $Q$ represents energy exchange between source and reactance of load
Summary

- RMS/Effective values: $V_{rms} = \frac{V_m}{\sqrt{2}}$, $I_{rms} = \frac{I_m}{\sqrt{2}}$
- Average Power $P = V_{rms}I_{rms}\cos(\theta_v - \theta_i)$ (Watts)
- Apparent Power $S = V_{rms}I_{rms}$ (VA)
- Power Factor (pf): $\cos(\theta_v - \theta_i)$
- Complex Power $S = \frac{V}{2}I^* = P + jQ$
- Reactive Power $Q = \frac{V_mI_m}{2}\sin(\theta_v - \theta_i)$
- Current Leads Voltage: Leading PF
- Current Lags Voltage: Lagging PF