16.202: Magnetically Coupled Circuits: Mutual Inductance

- **Background:**
- **Inductor:** Passive element that resists *change* in current flow. 
  \[ V_L = L \frac{di}{dt} \]
- Consists of \( N \) turns of wire wound around a core material with permeability \( \mu \);
- \( L = \frac{N^2 \mu A}{l} \); \( N \) : No. of Turns; \( A \) : Cross-Sectional Area; \( l \) : Length.
- **Amperes Law/ Biot-Savart:** Current flowing in the coil creates a magnetic field.
- A closed conducting loop carrying a current \( i \) when placed in an external magnetic field, experiences a force. (causes a torque/rotation) (Electric Motors)
• Symmetry Principle: (Faraday's Law of Induction):
  – Exerting a torque on a closed conducting loop in an external magnetic field, causes electric current to flow in the loop [Electric Generators]
  – Induced EMF (Electromotive Force) appears when the number of magnetic field lines passing through the loop are changing.

• Magnetic Flux: Number of Magnetic Lines that pass through a surface bounded by the closed loop.
Mutual Inductance

\[ i_1(t) \quad L_1 \quad + \quad \phi_{11} \quad \phi_{12} \quad - \quad v_1 \quad L_2 \quad + \quad \quad v_2 \quad - \]

\[ \phi_1 = \phi_{11} + \phi_{12} \]

- Self Inductances: \( L_1 \) and \( L_2 \) (\( N_1 \) and \( N_2 \)) turns.
- Current \( i_1(t) \) flowing through \( L_1 \) produces magnetic flux \( \phi_1 \) that consists of two components
  \[ \phi_{11} : \text{Links only } L_1 \]
  \[ \phi_{12} : \text{Links both coils.} \]

\[ \phi_1 = \phi_{11} + \phi_{12} \]
• $L_1$ and $L_2$ are physically separated, but magnetically coupled.
• Induced voltages: $v_1 = N_1 \frac{d\phi_1}{dt}$ and $v_2 = N_2 \frac{d\phi_{12}}{dt}$
• In terms of source current $i_1(t)$: $v_1 = N_1 \frac{d\phi_1}{di_1} \frac{di_1}{dt} = L_1 \frac{di_1}{dt}$

\[ v_2 = N_2 \frac{d\phi_{12}}{di_1} \frac{di_1}{dt} = M_{21} \frac{di_1}{dt} \]

Where $L_1 = N_1 \frac{d\phi_1}{di_1}$

Mutual Inductance $M_{21} = N_2 \frac{d\phi_{12}}{di_1}$

• $M_{21}$: Mut. Inductance of coil 2 with respect to coil 1. (H)
• Suffix 21 refers to induction in coil 2 due to current in coil 1
• Open-circuit mutual voltage (induced voltage): $v_2 = M_{21} \frac{di_1}{dt}$
General Model

- Finding polarity of mutual voltage $M\frac{di}{dt}$.
- Depends on orientation of coil windings: Use Dot Convention
- Position of dot at one end of inductor gives direction of magnetic flux if current enters at the dotted terminal
• If current enters dotted terminal of one coil, reference polarity of mutual voltage in the second coil is *positive* at the dotted terminal of second coil

• If current leaves dotted terminal of one coil, reference polarity of mutual voltage in the second coil is *negative* at the dotted terminal of the second coil

![Diagram showing mutual inductance and voltage relationship](image)

Figure 3:
\[ L = L_1 + L_2 + 2M \]

\[ L = L_1 + L_2 - 2M \]

Figure 4: