16.202: Passive Filters

- Filter is said to be passive if it contains passive elements \( R, L, C \) only
- Active Filters include Op-Amps and Transistors
- The ideal frequency response of filters:

\[
|H(\omega)|
\]

**Figure 1: Filter Characteristics**
• **PassBand**: Band of frequencies through which signals are passed from input to output

• **StopBand**: Band of frequencies not in the circuits passband

• Determine the frequency response of the filter $H(\omega)$ to plot the magnitude and phase plots to determine filter characteristic
Low-Pass Filters

![Low-Pass Filters Circuit Diagram](image)

Figure 2: Low Pass Filters

- Determine the limiting frequency characteristics:
  - \( \lim_{\omega \rightarrow 0} \)
    
    \[- R \rightarrow R \quad j\omega L \rightarrow 0 \text{(Short)} \quad -j/\omega C \rightarrow \infty \text{ (Open)} \]
  - \( \lim_{\omega \rightarrow \infty} \)
    
    \[- R \rightarrow R \quad j\omega L \rightarrow \infty \text{(Open)} \quad -j/\omega C \rightarrow 0 \text{ (Short)} \]
Series RL : Low Pass Filter

- Low freq: $\omega L << R$: Inductor : Short Circuit
- Voltage across the resistor follows the input voltage
- With increase in frequency: voltage drop across $L$ increases, reducing the output voltage
- High freq: $\omega L >> R$, Inductor : Open Circuit
- Output voltage goes to zero
- Cut-off Frequency: $\omega_c$: Frequency where $|H(\omega)| = H_{max}/\sqrt{2}$

$$H(\omega) = \frac{V_0}{V_i} = \frac{R}{R + j\omega L} = \frac{R/L}{R/L + j\omega}$$

$$= \frac{1}{1 + \frac{j\omega}{(R/L)}}$$
\( H(\omega) = \frac{1}{1 + \frac{j\omega}{R/L}} \quad |H(\omega)| = \frac{1}{\sqrt{1 + \frac{\omega^2}{(R/L)^2}}} \)

- \( |H_{max}| = 1 \) \quad Solve for \( \omega_c \):
- \( \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + \frac{\omega_c^2}{(R/L)^2}}} \)
- Cutoff Frequency: \( \omega_c = \frac{R}{L} \)

Figure 3: Low Pass: Series RL
• Example: Select values of $R$ and $L$ such that $f_c = 10$Hz.

• Select $L = 100$mH; Apply $\omega_c = R/L$

• $R = \omega_c L = 6.28$ Ohms

**Series RC Circuit: Low Pass**

• Output voltage is across the capacitor:

  $$H(\omega) = \frac{1}{1 + \frac{j\omega}{1/RC}}$$

• Cut-off Frequency: $\omega_c = 1/RC$

• Ex: If $f_c = 3$Khz required, Choose $C$ and determine $R$

• In the time-domain, the output voltage exhibits exponential decay with time constant: $\tau = 1/\omega_c$
High Pass Filters

Output voltage across $R$ in series RC: $H(\omega) = \frac{j\omega}{j\omega+1/RC}$

$\angle H(\omega) = 90 - \tan^{-1}\omega RC$  Cutoff freq: $\omega_c = 1/RC$
- Ex: Show that a series RL circuit behaves as a high-pass filter
- Find the cutoff freq. $\omega_c$

Example: Effect of Load Resistance:

![Circuit Diagram]

Figure 5: Effect of $R_l$

- Find transfer function $H(\omega)$
- $H(\omega) = \frac{K j\omega}{j\omega + K(R/L)}$
- where $K = \frac{R_L}{R+R_L}$ and $\omega_c = KR/L$
- Discuss the difference in filter characteristics
BandPass Filters

- Series and Parallel RLC Circuits
- Center Frequency $\omega_0$: $H(\omega_0)$ is real
- Cutoff Frequencies $\omega_1, \omega_2$ and Bandwidth $B = \omega_2 - \omega_1$
- Quality Factor $Q$. Measure $V_0$ across $R$. 
Band Reject Filters

- Series RLC: Measure output across \( L \) and \( C \):
- \( \omega = 0 \) and \( \omega = \infty \) Output = Input: Open circuit
- Two passbands: \( \omega < \omega_1 \) and \( \omega > \omega_2 \) \( X(\omega) = \omega L - 1/\omega C \)
- Increase \( \omega \): impedance of \( L \) increases and that of \( C \) decreases.
- \( \omega = \omega_0 \): \( \omega L = 1/\omega C \) Output voltage is zero (short circuit)