Attenuation and Backscatter From a Derived Two-Dimensional Duststorm Model

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Abstract—Fundamentals of attenuation and radar backscatter through duststorms are reviewed. A duststorm is modeled as circularly symmetric having a visibility with a minimum at its center (e.g., maximum mass loading) and which exponentially increases radially to a fixed maximum visibility threshold level (minimum mass loading). This model enables the convenient calculation of the two dimensional (2-D) structure of radar backscatter and path attenuation. As an example, the parameters of the exponential function describing the visibility distribution for a particular duststorm was derived using measurements made in the Sudan by other investigators operating a 10.5 GHz, 25 km link. A comparison of the calculated and measured attenuation time-series showed relatively close agreement. Both attenuation levels and backscatter levels due to even intense duststorms are expected to be relatively small for frequencies up to 10 GHz. For example, the peak attenuation for the duststorm that contained visibilities smaller than 2 m was less than 6.5 dB. Modeled backscatter due to this duststorm gave levels smaller than that obtained by an equivalent rainrate of 0.6 mm/h. Although the calculations were obtained for X-band, they may be extended to higher frequencies. Frequency scaling at 37 GHz, for example, showed a peak equivalent path attenuation level of at least 26.6 and 48 dB under varying assumptions.

Index Terms—Duststorm, duststorm RF attenuation, duststorm RF backscatter, duststorm visibility.

I. INTRODUCTION

ATTENUATION and scattering from duststorms have been examined by a number of investigators. Bashir and McEwan [1], in an excellent review of microwave propagation in duststorms, concluded that except in very rare instances attenuation due to absorption and scatter is negligible for frequencies up to 30 GHz. Calculations by Goldhirsh [2] using particle size distribution measurements of Ghobrial [3] demonstrated negligible attenuation at frequencies of X band and smaller frequencies for realistic values of dust particle concentrations extending over the path. Bashir and McEwan [1] also suggest that fading during duststorms along terrestrial links might possibly be attributed to effects due to cross polarization, refractivity variations (superrefraction, subrefraction, multipath), and dust accretion on reflector antennas.

An intent of this effort is to characterize the two-dimensional (2-D) structure of a severe duststorm in terms of its visibility parameters and relate the structure to the path attenuation and radar backscatter at X band and other frequencies. Although severe duststorms are rare, when they do occur, they can potentially result in severe mitigation of communications or introduce unwanted clutter to a radar system operating at higher frequencies. To the author’s knowledge, no 2-D model of the visibility structure of a duststorm has hitherto been developed. Knowledge of the structure enables the scaling of measured path attenuation and/or radar backscatter levels to other frequencies. Such information also provides information as to whether a backscatter from a hard target within a duststorm may be discriminated from the clutter. The structure of the duststorm also provides information as to the azimuthal dependence of both clutter and attenuation. Time series of the attenuation and clutter may also be modeled assuming the frozen duststorm advects through the path in question at a defined velocity.

The onset of duststorms has been described by investigators in terms of the onset of “an advancing wall which may be miles long and several thousand feet high” [4]. Joseph et al. [5] describe in-situ and radar measurements of duststorms in India called “Andhi.” These are convective storms that last on average from five to 20 min and sometimes have rainfall associated with them after the onset of the storm. Visibility records during the storm generally show rapid decrease of visibility (e.g., from 3 km to 120 m in three min) followed by an approximate exponential increase in visibility (e.g., from 120 to 1000 m in 28 min).

The methodology considered here is to characterize the visibility structure of duststorms from which the 2-D path attenuation and radar backscatter may be calculated. In this effort, the visibility of a duststorm is assumed to be circularly symmetric relative to its center where it is minimum. It increases exponentially in the radial direction up to a predefined threshold level. Example parameters associated with the exponential distribution of visibility are determined from terrestrial link measurements at 10.5 GHz for the case of a “severe” duststorm in Khartoum, Sudan[6]. Although the results are expressed in terms of a specifically measured severe duststorm, they may be generalized to other duststorms, providing a limited number of measurements of visibility and wind speed (described in this effort) are made. The exponential representation of a duststorm is inconsistent with the “advancing” wall appearance of duststorms. It, nevertheless, does allow computational ease and it is believed to give reasonable estimates of attenuation and backscatter.

II. PHYSICAL DESCRIPTION

A. Duststorms Versus Sandstorms

Duststorms are frequently mislabeled as “sandstorms.” These are two different phenomena. Duststorms usually occur over arable land where there has been a drought over extended periods. Strong winds may raise the fine surface particles, referred
to as dust, as high as one kilometer. The particle diameters are generally less than 80 μm and may be as small as 10 μm. Hence, the fall speeds of such particles are such that the dust may obscure the sun for extended periods. To be classified as a dust-storm, the visibility must be smaller than 1 km. When the visibility is shorter than 500 m, it is called a “severe duststorm” [4]. Ghobrial and Sharief [7] reported that the chemical composition of typical samples collected in Khartoum are by weight 62% silicon dioxide, and 38% other metallic oxides including iron and aluminum.

On the other hand, sandstorms generally refer to sand being driven by winds where the sand particles rarely rise higher than two meters [8]. The horizontal movement of sandstorms is usually through a process called “saltation.” This a phenomenon such that the sand particles are blown laterally by the wind and at the same time the particles bounce from the surface such as may occur with ping pong balls in a horizontal wind field. The diameters of most sandstorm particles are greater than 0.08 mm and are usually between 0.15 and 0.3 mm [8]. Sand particles are usually comprised of 92% silicon dioxide by weight [7]. Sandstorms are most likely to develop in desert regions where there is loose sand, and they tend to form during the day and die out at night [4].

### B. Frequency of Occurrence of Duststorms

As an example, Fig. 1 represents the average yearly conditional cumulative distribution of visibility averaged over a five-year period and combined over four site locations in Sudan (Abu Hamad, Atbara, Khartoum, Elobia). This curve was derived from calculations of tabulated data of hourly visibilities presented by Ghobriel and Sharief [7]. The ordinate represents the percentage of the average year the visibility is smaller than the abscissa conditioned to the existence of a duststorm; a duststorm. The average number of annual hours for the four sites in which the visibility was smaller than 1000 m was 64.7 h. Hence, the absolute probability of a duststorm is 0.74% for the case considered. The results of Fig. 1 show the occurrence of a duststorm with a visibility smaller than 50 m is approximately 1.3 h of the average year.

### III. Attenuation Due to Sand or Duststorms

The attenuation coefficient due to dust may be expressed by the formulation [2]

\[
k = \frac{1.029 \cdot 10^6 \cdot \varepsilon''}{[(\varepsilon' + 2)^2 + \varepsilon''^2]} \cdot \lambda \sum_i N_i \cdot r_i^2 \text{ [dB/km]}
\]  

where \(\varepsilon', \varepsilon''\) are the real and imaginary contributions of the relative dielectric constant of the dust particles, respectively, \(\lambda\) is the wavelength in meters, \(N_i\) is the number of particles of dust with radii between \(r_i\) and \(r_i + \Delta r_i\) per m\(^3\) and \(r_i\) is the dust radius in m belonging to the \(i\)th bin of a particle size distribution. The expression (1) is applicable at wavelengths for which the Rayleigh condition is applicable. That is

\[
\frac{\pi \cdot D}{\lambda} \ll 1
\]  

where \(D\) is the dust particle diameter. Assuming \(D_{\text{max}} = 0.2\) mm and the right hand side of (2) is 0.1 or smaller, the maximum applicable frequency for the above formulation is \(f_{\text{max}} \approx 48\) GHz.

The total relative volume of all dust particles per cubic meter of air may be expressed by

\[
v_r = \frac{4}{3} \pi \sum_i N_i r_i^3 \text{ [m}^3\text{ of dust/m}^3\text{ of air]},
\]  

Employing the form (3), the attenuation coefficient (1) may be reexpressed as

\[
k = \frac{2.457 \cdot 10^5 \cdot \varepsilon''}{[(\varepsilon' + 2)^2 + \varepsilon''^2]} \cdot \lambda \cdot v_r \text{ [dB/km]}
\]  

which is the same expression derived by Ghobriel and Sharief [7] by alternate means. We will return to the expression (4) shortly.

The “mass loading” \(M\) is defined as the relative mass of dust per cubic volume of air and is given by

\[
M = \rho \cdot v_r \text{ [kg of dust/m}^3\text{ of air]}
\]  

where the average measured density of nine samples collected in Khartoum, is given by \(\rho = 2.44 \times 10^3\) kg/m\(^3\) [7]. The visibility \(V\) (in kilometers) is related to the “mass loading” \(M\) (in kg/m\(^3\)) by the expression [9]

\[
M = \frac{C}{V\gamma} \text{ [kg/m}^3\text{]}
\]  

Although, different investigators use various values of \(C\) and \(\gamma\), applicable values for the region of Sudan were found to be \(C = 2.3 \cdot 10^{-5}\), \(\gamma = 1.07\) [7], [10]. These values are consistent.
with those reported by the National Center for Atmospheric Research [10].

Substituting the above values of $C$ and $\gamma$ into (6) and dividing the resultant expression by the value of $\rho$ gives the following expression for the total relative volume

$$ v_r = \frac{M}{\rho} = \frac{9.43 \times 10^{-9}}{V^\gamma} \quad [\text{m}^3 \text{of dust/m}^3 \text{of air}], \quad (7) $$

The attenuation coefficient may now be expressed in terms of the visibility by substituting (7) into (4)

$$ k = \frac{2.317 \times 10^{-3} \cdot e''}{[(e' + 2\gamma)^2 + e''^2]^{0.5}} \cdot \frac{1}{V^\gamma} \quad [\text{dB/km}], \quad (8) $$

Ghobrial and Sharief [7] measured $e' = 5.23, \epsilon'' = 0.57$ (4% water) and $e' = 5.23, \epsilon'' = 0.26$ (dry dust) at approximately 10 GHz. We assume that the 2% water content permittivities are obtained by averaging the above respective real and imaginary contributions, giving $e' = 5.733, \epsilon'' = 0.415$ (2% water). Substituting these values of permittivity into (8) results in

$$ k = W \left(\frac{1}{\lambda}\right) \left(\frac{1}{V^\lambda e''}\right) \quad [\text{dB/km}] \quad (9) $$

where $\lambda$ is the wavelength in meters, $V$ is the visibility in kilometers, and $W = 1.94 \times 10^{-5}$ (4% water), $W = 1.60 \times 10^{-5}$ (2% water), and $W = 1.15 \times 10^{-5}$ (0% water). The expression (9) applies to frequencies at or in the vicinity of X band since the calculation of $W$ is based on dielectric constants measured in this band.

IV. 2-D CHARACTERISTICS OF A DUSTSTORM

A. Visibility and Mass Loading

As a simplification to the characteristic visibility associated with duststorms, we approximate this visibility by the 2-D exponential formulation

$$ V = V_o \cdot \exp \left( -\frac{r}{r_o} \right) \quad (10) $$

where $r$ is the distance from the center of the dust storm, $V_o$ is the minimum visibility at $r = 0$, and $r_o$ is the characteristic radial distance over which the minimum visibility increases by a factor of $\exp(1)$.

As an example, we estimate $V_o$ and $r_o$ using the measured data from the duststorm examined by Ghobrial and Jervase [6] on May 9, 1990. They reported wind speeds as high as 30 kn (15.4 m/s) during the storm. Individuals observed visibilities in the range 2 to 5 m for periods lasting for more than 20 min. Observers at Khartoum airport reported visibilities smaller than 100 m for a period of 80 min. Assuming the exponential shape of the sandstorm is frozen in space and the storm peak advected past the observers with the measured wind speed of 15.4 m/s, it may be deduced from the above observations that visibilities smaller than 5 m and 100 m result in respectively radial distances relative to the minimum visibility levels of 9.25 km and 37 km, respectively (Fig. 2). These values of radial distances and corresponding visibilities when substituted into (10) enable a determination of estimated values of $V_o$ and $r_o$, namely,

![Fig. 2. Illustrative example showing distance intervals over which observed visibilities were smaller than indicated levels assuming the visibility varies exponentially with distance.](image)

![Fig. 3. Visibility and mass loading versus distance from center consistent with the duststorm in Khartoum, on April 9, 1991.](image)

$$ V_o = 1.84 \times 10^{-3} \quad \text{km} \quad \text{and} \quad r_o = 9.26 \quad \text{km}. \quad \text{In Fig. 3 is given the spatial variability of visibility of the duststorm in Khartoum, on April 9, 1991 consistent with (10) and the above values of $V_o$ and $r_o$.} $$

The mass loading $M$ may alternately be expressed as

$$ M = \rho v_r = \frac{4}{3} \cdot \pi \cdot \rho \sum N_i \cdot r_i^2 \quad [\text{kg/m}^3]. \quad (11) $$

Substituting the above value of $V_o$ and $r_o$ into (10) and the resultant expression into (6), the mass loading may alternately be expressed by

$$ M = 1.94 \times 10^{-2} \cdot \exp(-0.116 \cdot r) \quad [\text{kg/m}^3]. \quad (12) $$
This expression is plotted in Fig. 3 (scale on right side). The peak mass loading occurs at \( r = 0 \) and is 19.4 g m\(^{-3}\) of air. At 40 km from the peak value, it reduces to approximately 0.2 g/m\(^{3}\).

**B. Attenuation Coefficient**

In Section V, it is demonstrated that the attenuation coefficient (9) derived using the permittivity corresponding to 2% water content of dust agreed best with the measured results of Ghobrial and Jervase [6]. Substitute (10) into (9) using \( W = 1.60 \times 10^{-5}, V_o = 1.84 \times 10^{-3} \text{ km}, \) and \( r_o = 9.26 \text{ km}, \) we obtain the expression for the attenuation coefficient for dust particles having 2% water. That is

\[
k(x, y) = \frac{1.35 \times 10^{-2}}{\lambda} \exp\left(-0.116 \cdot r\right) \text{ [dB/km]} \tag{13}
\]

where \( \lambda \) is expressed in meters. As an illustration, the attenuation coefficient versus distance from the minimum visibility location is plotted in Fig. 4 (right scale) for the link frequency of 10.5 GHz used by them. The peak attenuation coefficient is approximately 0.47 dB/km at \( r = 0 \) km and at 40 km it becomes 4.6 \( \times \) 10\(^{-3}\) dB/km.

**V. CONSTRUCTING 2-D DUST STORM FIELD**

**A. 2-D Visibility and Attenuation Coefficient**

In this section is constructed the above described duststorm-relative to the center of a rectangular grid with dimensions 200 km \( \times \) 200 km comprised of sub-grids of 0.5 km \( \times \) 0.5 km. We assume the minimum visibility occurs at the grid location \( x_o = 100.25 \) km and \( y_o = 100.25 \) km. The visibility, as given by (10) referenced to the coordinate location \( (x_o, y_o) \) and evaluated at the center of each of the subgrid locations denoted by \( (x_i, y_i) \) may be expressed by

\[
V(x_i, y_i) = V_o \cdot \exp\left[\frac{\left((x_i - x_o)^2 + (y_i - y_o)^2\right)^{1/2}}{r_o}\right]. \tag{14}
\]

The expression (14) has been evaluated at the center of each of the subgrid locations employing the above values of \( V_o \) and \( r_o \) throughout the above-described 100 km \( \times \) 100 km area. Fig. 5 shows the corresponding 2-D visibility field that is capped at a visibility of 30 km. The vertical scale is expressed as the \( \text{Log}_{10} \) of the visibility in meters.

The 2-D attenuation coefficient field may analogously be constructed from (13) and is represented by

\[
k(x, y) = \frac{1.35 \times 10^{-2}}{\lambda} \exp\left(-0.116 \cdot r\right) \text{ [dB/km]} \tag{15}
\]

where \( x_o, y_o, x_i, y_i \) are defined above.

**B. Comparison With Measured Data**

We examine here the path attenuation at 10.5 GHz observed on a 25-km link in the vicinity of Khartoum by Ghobrial and Jervase [6]. Vertically polarized signals were transmitted at a height of 30 m and the receive antenna was at a height of 27 m. Although a large number of dust storms were encountered during a four-month period that started on March 15, 1990, most were mild ones with visibilities exceeding 500 m and showed no measurable attenuation. The duststorm of May 9, however, produced a visibility smaller than 5 m and resulted in a peak attenuation of 6.4 dB. The time series of the attenuation measured by them is shown in Fig. 6 (solid curve). Since this time series was extracted from their paper using a digitizer, small differences with the measured results are expected (smaller than 0.5 dB).

**C. Modeling the Attenuation Time Series**

Ghobrial and Jervase [6] reported the storm front relative to the propagation path was approximately 45° as depicted in Fig. 7. A code was developed which simulates the advection of the attenuation coefficient field given by (15) in the direction shown in Fig. 7. In simulating the attenuation time series, the following was assumed: 1) Movement of the field was taken in 0.5 km intervals. 2) The speed of the duststorm was assumed to be 30 knots (56 km/h), consistent with that measured by Ghobrial and Jervase [6]. 3) The attenuation coefficient was assumed constant over each subgrid and assumed to have the value calculated using (15). 4) The attenuation for each instant of time was calculated by multiplying the attenuation coefficient values at the center of each subgrid by the path length interval value 0.707 km that corresponds to the diagonal length through the subgrid. The resultant time series of the path attenuation is given by the dashed curve in Fig. 6. Since the relative reference time is somewhat arbitrary, the peak value of the modeled time series was matched with that of the measured levels. The modeled
time series is shown to agree relatively well with that measured, especially in capturing the value of the peak attenuation.

D. Peak Path Attenuations at Different Frequencies

As mentioned, the expression (15) is applicable at or in the vicinity of $X$ band since the values of dielectric constant from which the indicated multiplying coefficient was derived was also evaluated in this band. Ansari and Evans [11] provide a tabulation of real and imaginary parts of the dielectric constant as a function of frequency (from 3 to 37 GHz) for different soil types measured by various investigators. Table I gives a sampling of the dielectric constants and moisture concentrations measured at the indicated frequencies. Also listed are corresponding calculated peak attenuation coefficients, defined as $k(t_{\text{max}})$, which multiply the exponent in (15) and the peak path attenuations $A(t_{\text{max}})$ scaled from the $X$ band path described here. In applying these values, several caveats must be borne in mind. First, the moisture concentration of the soil is 5% at 14, 24, and 37 GHz, whereas the soil moisture at 3 and 10.5 GHz is approximately 2%. Secondly, the soil types are different consisting of loam, clay, silt and sand ascribed to the different investigators. The peak attenuation values are plotted as a function of frequency in Fig. 8 (dashed curve). Also plotted are the calculated peak attenuations assuming the dielectric constant is the same as described for the $X$ band path (i.e., the permittivity is independent of frequency). Under the latter assumption, the peak path attenuation would reach a level of 22.6 dB at 37 GHz.

VI. RADAR CLUTTER FROM DUSTSTORMS

The visibility of the dust field of the type given in Fig. 5 may be used to obtain estimates of expected radar backscatter. In this section is derived an expression of the received power in terms of the visibility. As an example, an estimate of the received power profile is derived assuming the visibility structure
TABLE I

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>Moisture (%)</th>
<th>Soil Type</th>
<th>ε'</th>
<th>ε''</th>
<th>k(max) (dB/km)</th>
<th>A(max) (dB)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2.2</td>
<td>loam</td>
<td>3.5</td>
<td>0.14</td>
<td>0.00905</td>
<td>1.2</td>
<td>Van Hippel [12]</td>
</tr>
<tr>
<td>10.5</td>
<td>2</td>
<td>clay, silt</td>
<td>5.73</td>
<td>0.415</td>
<td>0.474</td>
<td>6.4</td>
<td>Ghobrial and Sharief [7]</td>
</tr>
<tr>
<td>14</td>
<td>5</td>
<td>sand</td>
<td>3.9</td>
<td>0.62</td>
<td>1.609</td>
<td>21.7</td>
<td>Njoku and Kong [13]</td>
</tr>
<tr>
<td>24</td>
<td>5</td>
<td>sand</td>
<td>3.8</td>
<td>0.65</td>
<td>2.987</td>
<td>40.3</td>
<td>Njoku and Kong [13]</td>
</tr>
<tr>
<td>37</td>
<td>5</td>
<td>loam</td>
<td>2.88</td>
<td>0.3529</td>
<td>3.558</td>
<td>48.0</td>
<td>Geiger and Williams [14]</td>
</tr>
</tbody>
</table>

Fig. 8. Equivalent path attenuation versus frequency using dielectric constants of Table I and fixed dielectric constant of Ghobrial and Sharief [7].

The power received at a pulsed radar from distributed targets such as raindrops is similar to that received from dust as suggested by Goldhirsh [2]. The power, referenced to the antenna gain measurement point, is given by

\[ P_r = \frac{c}{1024 \times \pi^2 \ln 2} \left[ P_t \times \tau \times C^2 \times (\Delta \theta_v) \times (\Delta \theta_h) \times \eta \times R \right] \times 10^{-\frac{1}{10}(k_d + k_g + k_v)} \]

(16)

where \( c \) is the speed of light, \( P_t \) the transmitted power (W), \( \tau \) the pulsewidth (s), \( \lambda \) the wavelength (m), \( C \) the antenna gain, \( \Delta \theta_v, \Delta \theta_h \) the vertical and horizontal beamwidths (radians), respectively, \( \eta \) the radar reflectivity of dust (m\(^{-1}\)), \( R \) the range to center of pulse volume (m), \( k_d, k_g, k_v \) attenuation coefficients due to dust particles and gaseous absorption, respectively (dB/km), and \( \ell \) is the path length through atmosphere (km).

It may be demonstrated that the radar reflectivity (radar cross section per unit volume) for a distributed target is given by [16]

\[ \eta = \frac{\pi^5}{\lambda^4} \cdot |K|^2 \cdot Z \quad [\text{m}^{-1}] \]

(17)

where

\[ |K|^2 = \frac{(\varepsilon' - 1)^2 + \varepsilon''^2}{(\varepsilon' + 2)^2 + \varepsilon''^2} \]

(18)

Assuming the previously determined values of \( \varepsilon' \) and \( \varepsilon'' \) for 2% moisture content (at X band), we obtain \(|K|^2 \approx 0.38\). Since the permittivities between 3 and 10 GHz are relatively constant at low soil moisture contents [13], \(|K|^2 \) may be assumed relatively invariant within this frequency interval.

The quantity \( Z \) is called the radar reflectivity factor given by

\[ Z = \sum_i D_i^6 \cdot N_i(D) \cdot \Delta D \]

(19)

where \( N_i(D) \Delta(D) \) is the number of dust particles whose diameters are between \( D_i \) and \( D_i + \Delta D \) per unit volume. The increment \( \Delta D \) is the diameter bin size of the measurement system. The above formulation is applicable at frequencies satisfying the Rayleigh condition (2). Multiplying and dividing (19) by \( N_T \), which is defined as the total number of dust particles per unit volume, we obtain

\[ Z = N_T \sum_i D_i^6 \cdot P_i \quad [\text{m}^3] \]

(20)

where \( P_i \) is interpreted as the probability that the dust particle has a diameter between \( D_i \) and \( D_i + \Delta D \) per unit volume. Substituting (20) into (17) and the resultant expression into (16), the received radar power becomes

\[ P_r = 1.311 \cdot 10^7 \left[ P_t \times \tau \times C^2 \times (\Delta \theta_v) \times (\Delta \theta_h) \times \eta \times R \right] \times 10^{-\frac{1}{10}(k_d + k_g + k_v)} \times \frac{|K|^2 \cdot N_T}{R^2} \sum_i D_i^6 \cdot P_i \]

(21)
where the attenuation factors due to gas and dust particles has been omitted to obtain an upper limit of the radar backscatter power.

The total number of particles per unit volume \( N_T \) is here expressed in terms of the mass loading \( M \). The mass loading given by (11) may alternatively be given by

\[
M = \frac{4}{3} \pi \rho \sum_i N_i \cdot \tau_i^3 = \frac{4}{3} \pi \rho N_T \cdot \sum_i P_i \cdot \tau_i^3. \tag{22}
\]

Substituting into the left-hand side of (22) \( M \) given by (6), \( \rho = 2.44 \times 10^3 \text{ kg/m}^3 \) and solving for \( N_T \), we obtain

\[
N_T = 2.25 \cdot 10^{-9} \cdot \frac{1}{V_{\text{1.07}}} \cdot \sum_i P_i \cdot \tau_i^3. \tag{23}
\]

Estimates of \( \sum_i P_i \cdot \tau_i^3 \) and \( \sum_i P_i \cdot D_i^6 \) have been evaluated by Goldhirsh [2] based on particle size distributions measurements by Ghobrial [3] and are given by

\[
\left( \sum_i P_i \cdot \tau_i^3 \right)_{\text{avg,max}} = 3.48 \cdot 10^{-14}, 5.57 \cdot 10^{-14} \text{ [m}^3]\tag{24}
\]

\[
\left( \sum_i P_i \cdot D_i^6 \right)_{\text{avg,max}} = 1.51 \cdot 10^{-24}, 3.22 \cdot 10^{-24} \text{ [m}^6].\tag{25}
\]

Substituting (23) into (20), the radar reflectivity factor \( Z \) is given by

\[
Z = \frac{2.25 \cdot 10^{-9}}{V_{\text{1.07}}} \cdot \sum_i \frac{P_i \cdot \tau_i^3}{\sum_i P_i}. \tag{26}
\]

Substituting the visibility \( V \) given by (10) into (26), the radar reflectivity factor may be expressed in terms of the distance \( r \) from the center of dust storm. That is

\[
Z = \frac{2.25 \cdot 10^{-9} \cdot \exp\left(\frac{-1.07}{r_{\text{\(\alpha\)}}}\right)}{V_{\text{0.07}}^{\frac{1}{V_{\text{1.07}}}}} \cdot \sum_i \frac{D_i^6 \cdot P_i}{\sum_i P_i \cdot \tau_i^3}. \tag{27}
\]

Using the example values of \( V_0 \) and \( r_{\alpha} \) given above, (27) becomes

\[
Z = 1.90 \cdot 10^{-6} \cdot \exp\left(-0.116 \cdot r\right) \cdot \sum_i \frac{D_i^6 \cdot P_i}{\sum_i P_i \cdot \tau_i^3}. \tag{28}
\]

Substituting (24) and (25) into (28)

\[
Z_{\text{avg,max}} = 824.4 \exp\left(-0.160 r\right),
\]

\[
\times 110 \exp\left(-0.116 r\right) \text{ [mm}^6/\text{m}^3]. \tag{29}
\]

### A. Equivalence With Rainrate

We here examine the equivalence between the above peak values of the radar reflectivity factor and the rainrate giving the same value. Assuming a Marshall–Palmer raindrop size distribution [17], the rainrate \( RR \text{(mm/h)} \) in terms of the radar reflectivity factor \( Z \text{(mm}^6/\text{m}^3) \) is given by

\[
RR = \left( \frac{Z}{200} \right)^{0.625} \text{ [mm/h]}, \tag{30}
\]

### TABLE II

<table>
<thead>
<tr>
<th>Radar Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latitude</td>
<td>37° 51' 16.88&quot;</td>
</tr>
<tr>
<td>Longitude</td>
<td>75° 30' 48.4&quot;</td>
</tr>
<tr>
<td>Peak power, ( P_r ) (W)</td>
<td>10^6</td>
</tr>
<tr>
<td>Center frequency (GHz)</td>
<td>2.84</td>
</tr>
<tr>
<td>Wavelength, ( \lambda ) (m)</td>
<td>0.106</td>
</tr>
<tr>
<td>Gain, G (value, dB)</td>
<td>1.148 x 10^7, (50.6)</td>
</tr>
<tr>
<td>Beamwidths ( \Delta \theta_h ) (radians, deg)</td>
<td>6.981 x 10^7, (0.4)</td>
</tr>
<tr>
<td>Pulselwidth, ( \tau ) (s)</td>
<td>10^4</td>
</tr>
<tr>
<td>Minimum detectable power (dBm)</td>
<td>-115</td>
</tr>
</tbody>
</table>

Substituting (29) into (30) we obtain the equivalent rainrates that give the same radar reflectivity factors. The peak values of these rainrates (at \( r = 0 \)) are

\[
(RR_{\text{avg,max}}) = 0.6, 0.7 \text{ [mm/h]}, \tag{31}
\]

These rainrates are considered very small and are at the level of “drizzle.”

Substituting (23) into (21), the received radar power may be expressed as follows:

\[
P_r = 2.95 \cdot 10^{-2} \cdot \left[ P_i \cdot \tau \cdot G^2 \cdot (\Delta \theta_h) \cdot (\Delta \theta_h) \cdot \lambda^2 \right] \cdot \frac{\left[ K \right]^2 \cdot 1}{V_{\text{1.07}}} \cdot \sum_i P_i \cdot D_i^6 \cdot \tau_i^3. \tag{32}
\]

As an example, we assume the parameter values of a meteorological radar located at the NASA Wallops Flight Facility at Wallops Island, VA. This radar, called SPANDAR [15], operates at S-band (\( f \approx 3 \text{ GHz} \)) and nominal parameter values are summarized in Table II. Substituting these values into (32) along with (10), (24), and (25), we obtain for the average received radar power

\[
P_r(S\text{-band}) = 2.35 \cdot 10^{-2} \cdot \frac{\exp\left(-0.116 \cdot r\right)}{R^2} \text{ [W]}, \tag{33}
\]

where \( R \) and \( r \) are expressed in meters and in kilometers, respectively. Assuming the center of the duststorm is at \((x_0, y_0)\) relative to the radar location (origin of coordinate system) and the radar pulse volume is centered at \((x_3, y_3)\), the radar power is given by (34) at the bottom of the next page, where the units in the exponent and the numerator are both in kilometers. Assuming all the parameter values in the square bracket of (32) are maintained constant except the wavelength, and the radar power (34) may be converted to other frequencies by noting

\[
P_r(f) = P_r(S\text{-band}) \cdot \left( \frac{f}{2.84} \right)^2 \cdot \frac{[K(f)]^2}{[K(2.84)]^2} \text{ [W]} \tag{35}
\]
Fig. 9. Plot of $S$- and $X$-band radar powers along a diagonal that cuts the minimum visibility location of the duststorm (solid curve) for example radar systems. The curve labeled “equivalent rainrate” (right scale) gives rainrates that produce the same power levels as the dust particles.

where $f$ is the frequency in GHz. Radar powers at $S$ (2.84 GHz) and $X$ (10 GHz) bands are plotted in Fig. 9 for the example radar described in Table II assuming the ratio of $|K|^2$ in (35) is unity. Equation (35) may be modified to frequencies greater than $X$ band (up to 48 GHz) by substituting appropriate permittivities (e.g., given in Table I) and moisture contents into $|K|^2$. The plots in Fig. 9 correspond to a scenario in which the above-described radars are located in proximity of the peak dust concentration of the duststorm described by (10). In particular, the location of the minimum visibility of the sandstorm is assumed to be at $x_0 = 40.25$ km, $y_0 = 40.25$ km relative to the radars located at $(0,0)$. As in the previous example, we assume an area comprised of subgrids of 0.5 km $\times$ 0.5 km and evaluate the powers at the center of these grids. The radar powers in Fig. 9 are evaluated along a diagonal that passes through the grid in which the visibility is minimum. The power is shown from a range of 10 km out to a range where the receiver noise level reduces to approximately $-115$ dBm (threshold noise level). Also shown plotted is the “equivalent rainrate” in mm/h (right-hand scale) defined as that rainrate that gives the same radar power as the dust particles. It is noted that at $S$- and $X$ bands, the locations at which backscatter are measured are limited to approximately 110 and 130 km, respectively. In the next section, it is demonstrated that the finite beamwidth at extended ranges is likely to be only partially filled because of the limiting height of the duststorm. Beyond these threshold ranges, the backscatter will be smaller than the values indicated in Fig. 9.

B. Mitigation of Backscatter Effects Due to Limiting Height of Duststorm

In Fig. 10 is given a family of curves describing the height of the upper edge of the pulse volume for different beamwidths as a function of the range. Hence, assuming a duststorm height of one kilometer, radars with beamwidths of $2^\circ$, $1.0^\circ$, and $0.5^\circ$ will have associated pulse volumes that are partially filled at ranges greater than approximately 30, 55, and 85 km, respectively. Hence, the backscatter power will be smaller than the indicated power in Fig. 9 (beamwidth = 0.5$^\circ$) at ranges larger than 85 km. The heights of severe dust storms are variable and have generally been reported to be less than 1.5 km from the surface. For example, Lawson [18] reported the height of a “haboob” (Sudanese term for a duststorm raised by strong winds) to be 1.2 km. Sutton [19] reported that the dust associated with a haboob is frequently raised to a height over 1 km. A nominal height of one kilometer is considered reasonable.

VII. CONCLUSION

The visibility structure of a duststorm has been modeled by a circularly symmetric 2-D exponential function (10) having two unknown parameters. These are the minimum visibility $V_o$.

$$P_v(S\text{-Band}) = 2.35 \times 10^{-8} \cdot \exp\left\{-0.116 \cdot \frac{\left[(x_0 - x_1)^2 + (y_0 - y_1)^2\right]^{1/2}}{(x_1^2 + y_1^2)}\right\}$$

(W).

$$P_v(X\text{-Band}) = 2.35 \times 10^{-8} \cdot \exp\left\{-0.116 \cdot \frac{\left[(x_0 - x_1)^2 + (y_0 - y_1)^2\right]^{1/2}}{(x_1^2 + y_1^2)}\right\}$$

(W).

$$P_v(X\text{-Band}) = 2.35 \times 10^{-8} \cdot \exp\left\{-0.116 \cdot \frac{\left[(x_0 - x_1)^2 + (y_0 - y_1)^2\right]^{1/2}}{(x_1^2 + y_1^2)}\right\}$$

(W).
and the characteristic radial distance $r_o$, where the visibility increases by a factor $\exp(+1)$ relative to the minimum value up to a predefined threshold value (e.g., 30 km). The visibility model enables the construction of a 2-D visibility field from which the path attenuation and radar backscatter may be characterized.

A measurement of the visibility at two locations in the duststorm enables a determination of the unknown exponential parameters $V_o$ and $r_o$. This formulation was applied to an intense duststorm in the Sudan reported by Ghobrial and Jervase [6] who operated a 10.5-GHz, 25-km link. The modeled attenuation time series gave very good agreement with the measured values (Fig. 6). In spite of the fact that this particular duststorm was severe with a minimum visibility of less than 2 m, the peak attenuation was less than 6.5 dB, and the peak radar reflectivity factor was smaller than obtained from an equivalent rainrate of 0.6 mm/h (Fig. 9). This rainrate is typical of light drizzle. The radar backscatter is further mitigated at extended ranges because the pulse volume may only be partially filled with dust due to the limiting height of the duststorm that is nominally 1 km (Fig. 10).

Scaling the peak path attenuation as a function of frequency, a value of 22.6 dB was calculated at a frequency of 37 GHz assuming the dielectric constant used for the path is independent of frequency. Using the measured dielectric constant at 37 GHz reported by Geiger and Williams [14], a peak path attenuation of 48 dB was calculated. The difference in the soil type (loam) and moisture content (5%) is, in part, attributed to the large difference in calculated values.

A major caveat in implementing the above-described model is that the visibility structure of an actual duststorm is such that the advancing front end is generally more abrupt than the back end. In fact, duststorms of the type encountered in the Sudan has been characterized as a moving wall of dust. Although the actual 2-D structure of visibility may deviate from the modeled exponential, the model is believed to be a convenient analytical tool that provides reasonable estimates of the attenuation and backscatter field. Additional comparisons between modeled and measured physical parameters such as path attenuation and backscatter are required to reinforce the validity of the model.

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REFERENCES


