University of Massachusetts Lowell
Department of Electrical and Computer Engineering
16.520 Computer Aided Engineering Analysis
Problem Set 7

The purpose of this exercise is to determine the impulse response of a filter which will be used in QMF bank. For the M order filter bank the cutoff frequency of the basic filter is $\omega_s = \pi/M$. The impulse response is given by $h(n)$ for $n = (0, N-1)$. The function h is symmetric. Therefore $h\left(\frac{N}{2} - 1 + j\right) = h\left(\frac{N}{2} - j\right)$ where $j = (1, N/2 - 1)$.

The amplitude of the frequency response given in Eqn 4 in the reference can be rewritten as

$$A(\omega) = 2 \sum_{i=0}^{\frac{N}{2}-1} h(i)\phi_i(\omega)$$

where

$$\phi_i(\omega) = \cos(\omega(\frac{N-1}{2} - i))$$

The energy in the stop-band (ω_s, π) of the filter is

$$E_{stop} = \frac{1}{\pi - \omega_s} \int_{\omega_s}^{\pi} A^2 d\omega = \sum_{i=0}^{\frac{N}{2}-1} \sum_{j=0}^{\frac{N}{2}-1} h(i)h(j)\left(\frac{4}{\pi - \omega_s} \int_{\omega_s}^{\pi} \phi_i(\omega)\phi_j(\omega)d\omega\right) = \underline{p}^T Q \underline{p}$$

where $\underline{p} = [h(0), h(1), \cdots, h(N/2-1)]^T$. The element Q_{ij} is equal to the bracketed term in the aforementioned equation. The matrix Q is symmetric.

The approach for finding h is to minimize E_{stop} subject to the constraint

$$4 \sum_{j=0}^{\frac{N}{2}-1-Mk} h(j+2Mk)h(j) = \delta(k)$$

for $k = (0, \textit{integer part of } (N-1)/(2M))$. When N and M are powers of 2 the integer part of $(N-1)/(2M) = N/(2M)$. In matrix form the aforementioned equation can be written as

$$G\underline{p} = \underline{d}$$

where $G_{kj} = h(j + 2Mk)$ for $j = (0, N/2 - 1 - Mk)$ and zero otherwise. The vector $\underline{d} = [1, 0 \cdots, 0]^T$.

Using the Lagrange multiplier approach yields

$$L = \underline{p}^T Q \underline{p} + \underline{\lambda}^T \left(G \underline{p} - \underline{d} \right)$$

The appropriate equations are obtained by minimizing L. Hence

$$\frac{\partial L}{\partial \underline{p}} = 2Q\underline{p} + G^T \underline{\lambda} = 0$$

$$\frac{\partial L}{\partial \underline{\lambda}} = G\underline{p} - \underline{d} = 0$$

a) Given that $M = 16$ and $N = 256$ find h using the iterative procedure outlined in the reference. The matrix is ill-conditioned however *gess* can be used in the calculation. $\omega_s = \pi / M$.