University of Massachusetts Lowell
Department of Electrical and Computer Engineering
16.520 Computer Aided Engineering Analysis
Problem Set 3

1. Consider the system

$$\begin{bmatrix} 1 & 2 & 3 \\ 6 & 7 & 8 \\ 0 & 0 & \varepsilon \end{bmatrix} \dot{x} = \begin{bmatrix} 30 & 17 & 16 \\ 0 & 3 & 6 \\ 12 & 1 & 4 \end{bmatrix} x$$

The parameter ε is equal 10^{-8}

a. Determine the eigenvalues and eigenvectors
b. Determine the stability of the system.
c. Evaluate the state transition matrix.

2. Given the transfer function

$$Y(s)/U(s) = 10s^2/(s^4 + 3s^3 + 2s^2 + s + 2)$$

a. Using the phase-variable decomposition method determine the impulse response of the system.

3. The function $f(x, y)$ satisfies the two-dimensional Laplace equation

$$f_{xx} + f_{yy} = 0 \Rightarrow \frac{f(x_{i+1}, y_j) - 2f(x_i, y_j) + f(x_{i-1}, y_j)}{(\delta x)^2} + \frac{f(x_i, y_{j+1}) - 2f(x_i, y_j) + f(x_i, y_{j-1})}{(\delta y)^2} = 0$$

for $1/2 \geq (x, y) \geq 0$. The boundary conditions are

$$f(x = 0, y) = 0$$
$$f(x, y = 0) = 0$$
$$f(x = 1/2, y) = 200y$$
$$f(x, y = 1/2) = 200x$$

Applying the discretization

$$x_i = i * \delta x \quad where\ \delta x = 1/2/N \quad \text{and } i = 0, N$$
$$y_j = j * \delta y \quad where\ \delta y = 1/2/N \quad \text{and } j = 0, N$$

a. Find the solution for $f(x_i, y_j)$ for $N = 5$ and $N = 10$ using a direct solution method.
b. Find the solution using Jacobi and Gauss Siedel methods.
c. Comapare the rate of convergence and accuracy of the methods used.