(3)

10 1(a) $H = x^2 + u^2 2 + \lambda [6x + u]$

10 (b) $u_0 \Rightarrow \frac{\partial H}{\partial u} = 0 = 4u + \lambda \Rightarrow \boxed{u^0 = -\frac{\lambda}{4}}$

(c) $H^0 = x^2 + \left[-\frac{\lambda}{4}\right]^2 (2) + \lambda\left[6x - \frac{\lambda}{4}\right]$

10 (a) $\dot{x} = 6x - \frac{\lambda}{4}$

10 (b) $-\dot{\lambda} = \frac{\partial H^0}{\partial x} = 2x + 6\lambda$

$$\begin{bmatrix} \dot{x} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} 6 & -1/4 \\ -2 & -6 \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix}$$

eigen vector/value

$\cancel{\sigma_1 = 2^{3/2}\sqrt{73}}$ $\cancel{e_1 =}$ $\left[\begin{matrix} 1 \\ \end{matrix}\right.$

$\cancel{\sigma_2 = -\sigma_1}$

$\sigma = 2^{3/2}\sqrt{73}$ ; $E = \begin{bmatrix} 1 & 1 \\ \sigma + 24 & -\sigma + 24 \end{bmatrix}$ $e^{\Lambda t} = \begin{bmatrix} e^{-\frac{73}{2}t} & 0 \\ 0 & e^{\frac{73}{2}t} \end{bmatrix}$

10 (d) $\Phi = E e^{\Lambda t} E^{-1}$

2

(a) $E = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ \frac{3}{2} & 2 & 0 & 0 \\ \frac{3}{2} & 0 & 0 & 0 \end{bmatrix}$ $\quad E^{-1} = \begin{bmatrix} 0 & 0 & 0 & 2/3 \\ 0 & 0 & \frac{1}{2} & -1/2 \\ 0 & 1 & 0 & 0 \\ 1 & -1 & -\frac{1}{2} & 1/2 \end{bmatrix}$

20

$$e^{\Lambda t} = \begin{bmatrix} e^{-4t} & 0 & 0 & 0 \\ 0 & e^{-3t} & 0 & 0 \\ 0 & 0 & e^{-2t} & 0 \\ 0 & 0 & 0 & e^{-t} \end{bmatrix}$$

(b) $\Phi = E e^{\Lambda t} E^{-1} =$

20

$$= \begin{matrix} e^{-t} & e^{-2t} - e^{-t} & \dfrac{e^{-3t} - e^{-t}}{2} & \dfrac{-e^{-t}}{6} \dfrac{-e^{-3t}}{2} + \dfrac{2}{3} e^{-4t} \\ 0 & e^{-2t} & 0 & 0 \\ 0 & 0 & e^{-3t} & e^{-4t} - e^{-3t} \\ 0 & 0 & 0 & e^{-4t} \end{matrix}$$

(c)

10 $$\begin{bmatrix} 0 & 0 & 0 & 2/3 \\ 0 & 0 & \frac{1}{2} & -1/2 \\ 0 & 1 & 0 & 0 \\ 1 & -1 & -1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \neq 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

not controllable