test 2

1 (a) 10 $\quad \frac{Y}{X} = \frac{(4s+k)/s^2}{1+[(4s+k)/s^2][1/(s+2)]}$

$$\frac{Y}{X} = \frac{(4s+k)(s+2)}{s^2(s+2)+(4s+k)} = \frac{(4s+k)(s+2)}{s^3+2s^2+4s+k}$$

10 (b) s^3+2s^2+4s+k

10 (c)

s^3	1	4
s^2	2	k
s	$\frac{8-k}{2}$	
s^0	k	

$8-k>0 \Rightarrow k<8$

$k>0 \Rightarrow k>0$

Stable $8>k>0$

10 (d) **$k=8$**

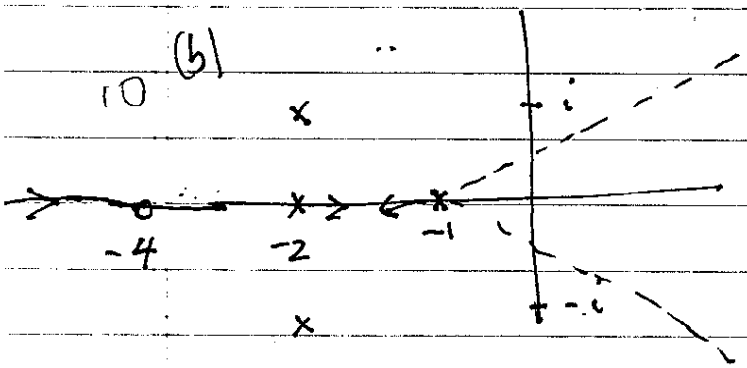
Aux poly $2s^2+k=0 \Rightarrow s = \pm j2$ (at $k=8$)

$f = \frac{2}{2\pi}$ hz

(2) (a) poles: $s = \pm i - 2$; zeros: -4

10 $s = -2, -1$

10 (b)



$$\sigma = \frac{(-7) + 4}{4 - 1} = \frac{-3}{3}$$

$$\phi = \frac{\pm(2n+1)\pi}{4-1} = \begin{cases} \pm\frac{\pi}{3} \\ \pi \end{cases}$$

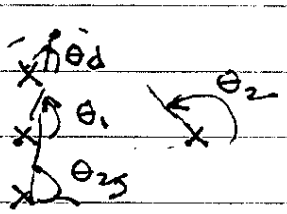
10 (c) $$k = -\left[\frac{s^4 + 7s^3 + 19s^2 + 23s + 10}{s+4}\right]$$

$$\frac{dk}{ds} = \frac{3s^4 + 30s^3 + 103s^2 + 152s + 82}{s^2 + 8s + 16} = 0$$

$s = -1.44$ breakaway pt

$k = 0.126$

10 (d) angle of dep.



$$\beta_1 + \theta_1 - \theta_2 - \theta_3 - \theta_d = \pm(2n+1)\pi$$

$\beta_1 = \tan^{-1}\left(\frac{1}{2}\right)$

$\theta_1 = \frac{\pi}{2}$

$\theta_3 = \frac{\pi}{2}$

$\theta_2 = \pi - \tan\left(\frac{1}{1}\right)$

3) 10

(a) $GH+1 = 1 + \dfrac{k}{s[s^2+6s+6]}$

$GH+1 = 0 = s[s^2+6s+6] + k = 0$

$s^3+6s^2+6s+k = 0$

given $\omega_n = 1$; $\zeta = 1/2$

then $s^2 + 2\omega_n\zeta s + \omega_n^2$ is a factor

$= s^2 + s + 1$

$(s^3+6s^2+6s+k) = (s+a)(s^2+s+1)$

$= s^3+s^2+s$
$\quad as^2+as+a$
$\Big\} \; s^3+s^2(1+a)+s(1+a)+a$

matching like terms in

$s^0 \quad k = a$

$s^1 \quad 1+a = 6 \Rightarrow \boxed{a=5}$

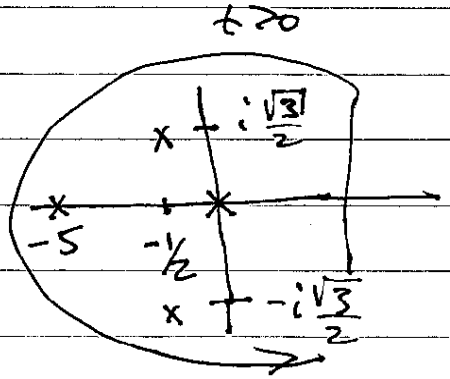
$s^2 \quad 1+a = 6$

$\Big\} \; \boxed{k=5}$

(3b)

$$\frac{Y}{X} = \frac{5}{(s+5)(s^2+s+1)}$$

$$Y = \frac{5}{s(s+5)(s^2+s+1)}$$



$t<0$

$y(t) = 0$

$t>0$

$$y(t) = Ye^{st}s\Big|_{s=0} + Ye^{st}(s+5)\Big|_{s=-5} + Ye^{st}\left(s+\tfrac{1}{2}-i\tfrac{\sqrt{3}}{2}\right)\Big|_{s=-\frac{1}{2}+i\frac{\sqrt{3}}{2}}$$

$$+ Ye^{st}\left(s+\tfrac{1}{2}+i\tfrac{\sqrt{3}}{2}\right)\Big|_{s=-\frac{1}{2}-i\frac{\sqrt{3}}{2}}$$

$$y(t) = e^{-t/2}\left\{-\frac{10}{7\sqrt{3}}\sin\left[\frac{\sqrt{3}}{2}t\right] - \frac{20}{21}\cos\left[\frac{\sqrt{3}}{2}t\right]\right\} - e^{-5t} + 1$$