1. Consider the coupled acoustic-mechanical system. The velocity of the masses are given by \( u \) and applied force by the variable \( f \). The variables \( k \) represent the mechanical stiffness, \( M \) the mass and \( b \) the damping coefficient. The closed open pipe is filled with a fluid having mass density \( \rho_0 \), sound speed \( c \), cross sectional area \( A \), length \( L \).

![Diagram of the coupled acoustic-mechanical system](image)

a. Using mobility analogy where the velocity as the "across" variable, determine the an equivalent circuit for the system.

b. Determine the equations of motion in the Laplace-domain.

c. Determine the equations of motion in the time-domain.

d. Find the transfer function \( \frac{U_2(s)}{U_o(s)} \).

1. Consider the a uncontrolled system where the open-loop transfer function is given by

\[
G(s) = \frac{(s + 6)}{s(s + 3)(s + 10)}
\]

and the input is \( X(s) \) and the output is \( Y(s) \).

a. Given the feedback gain is \( H(s) \) determine the transfer function \( \frac{Y(s)}{X(s)} \) for the negative feedback system.

b. If the error is defined as \( E = X - YH \) determine \( E/X \).

c. For \( x(t) = tu(t) \) find \( H(s) \) such that

\[
\lim_{t \to \infty} \frac{de}{dt} = \frac{1}{10}
\]
2. The error in a unity feedback system is the error $e(t) = x - y$ where $x$ is the input and $y$ is the output. The open loop-transfer function is

$$G(s) = \frac{5000}{s(s + 75)}$$

a. Determine the steady state error for $x = 5u(t)$
b. Determine the steady state error for $x = 5t^2u(t)$.

3. Given the block diagram shown below

![Block Diagram]

a. Determine its signal flow graph realization.
b. Using Mason’s gain formula determine $Y(s)/X(s)$.

4. Given the signal flow graph below determine the transfer matrix $A$ where $A_{ij} = Y_i/X_j$.

![Signal Flow Graph]

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

Note that $A_{ij} = Y_i/X_j$ given that all other inputs are equal to zero.
5. Given the system equations

\begin{align*}
\frac{dx_1}{dt} &= x_1 + 5x_2 \\
\frac{dx_2}{dt} &= 2x_1 + u
\end{align*}

a. Using only amplifiers and integrators draw a signal-flow graph representation of the system where \( U(s) \) is the input and \( X_1(s) \) is the output. You may assume zero initial conditions.

b. Find the transfer function \( X_1(s)/U(s) \) using Mason’s Gain formula. Check your result using an algebraic approach.