University of Massachusetts Lowell
Department of Electrical and Computer Engineering
16.413 Linear Feedback

Problem set 6

1. Consider the system

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -1 \end{bmatrix} \underline{x} + \begin{bmatrix} 4a & 2b \\ -4 & 2 \\ 0 & 0 \end{bmatrix} \underline{u}$$

$$\underline{y} = \begin{bmatrix} \alpha & 1 & 0 \\ 2 & \beta & 0 \end{bmatrix} \underline{x}$$

a. Find the eigenvalues and eigenvectors of the system.

b. Find the state transition matrix.

c. Determine when the system is controllable under the condition that a and b are real valued.

d. Determine when the system is observable under the condition the α and β are real valued.

2. Consider the transfer function for a causal system

$$\frac{Y}{U} = \frac{10(s+4)}{s^3 + 31s^2 + 230s + 200}$$

where U and Y are the Laplace transform of the input and output respectively. Using phase variables representation of the system matrices (A, B, C) where $\dot{\underline{x}} = A\underline{x} + B\underline{u}$ and $\underline{y} = C\underline{x}$.

a. Determine the state and output equations in terms of the state vector $\underline{x}$, input u and output y.

b. Find the state transition matrix.

c. If $u(t) = \delta(t)$ determine $y(t)$ using the STM.

3. Using the state function of Pontryagin minimize

$$PI = \int_0^1 \frac{1}{2}(x^2 + u^2)dt$$

subject to the constraint

$$\dot{x} = 10x + 4u$$

a. Find the state function of Pontryagin

b. Find the optimal input in terms of x and λ,

c. Find the givern equations for x and λ.

d. Find $x(t)$ and $u(t)$ given that $x(0) = 1$ and $x(1) = 2$.