HW 4

1.

a $$\frac{Y}{X} = \frac{G}{1+GH} = \frac{\frac{s+b}{s(s+a)}}{1+k\frac{(s+b)}{s(s+a)}}$$

(b) $$X - YH = E \Rightarrow \frac{E}{X} = 1 - H\frac{Y}{X} = 1 - \frac{GH}{1+GH} = \frac{1}{1+GH}$$

$$\frac{E}{X} = \frac{1}{1+GH} = \frac{1}{1+\frac{k(s+b)}{s(s+a)}}$$

(c) $$X(s) = \frac{3}{s} + \frac{1}{s^2} = \frac{3s+1}{s^2}$$

$$E = \frac{3s+1}{s^2\left[1+\frac{k(s+b)}{s(s+a)}\right]}$$

$$\lim_{s \to 0} sE = \frac{1}{\frac{kb}{a}} = e(\infty)$$

for $b=4a$ $$e(\infty) = \frac{1}{4k}$$

(d) $a<0$; $b>0$

char eqn $s^2 + s(k+a) + b$

$$\begin{array}{c|cc} s^2 & 1 & b \\ s & (k+a) & \\ s^0 & b & \end{array}$$

$\rightarrow k+a>0 \Rightarrow \boxed{k>-a}$

$\rightarrow b>0$ ✓

2. Sys 1

$$X_1(s+a) = bE \Rightarrow X_1 = \frac{b}{s+a}E$$

$$Y = CX_1 \Rightarrow \boxed{Y = C\frac{b}{s+a}E} \Rightarrow \frac{Y}{E} = \frac{cb}{s+a}$$

Sys 2

$$X_2(s+k) = lE \Rightarrow X_2 = \frac{lE}{s+k}$$

$$V = X_2 + mY \Rightarrow \boxed{V = \frac{lE}{s+k} + mY}$$

(a) $$\frac{V}{Y} = \frac{l}{s+k}\frac{E}{Y} + m = \cancel{cbl}$$

$$\frac{V}{Y} = \frac{l}{cb}\left[\frac{s+a}{s+k}\right] + m$$

$$\frac{Y}{E} = \frac{cb}{s+a}$$

$$\frac{Y}{W} = \frac{\left(\frac{Y}{E}\right)}{1+\left(\frac{V}{Y}\right)\left(\frac{Y}{E}\right)} = \frac{bc(s+k)}{s^2 + [bcm + l + k + a]s + bckm + al + ak}$$

(b)

$$\begin{array}{l|ll}
s^2 & 1 & bckm + al + ak \\
s^1 & [bcm + l + k + a] & \longrightarrow > 0 \\
s^0 & [bckm + al + ak] & \longrightarrow > 0
\end{array}$$

for stability

3

(a) $\dfrac{Y}{X} = \dfrac{GH}{1+GH}$

(b) $1+GH=0 \Rightarrow 1+\dfrac{k(s+5)}{s(s+1)(s+2)}=0$

$$s^3+3s^2+(k+2)s+5k$$

(c)

s^3	1	$k+2$
s^2	3	$5k$
s^1	$\dfrac{3(k+2)-5k}{3}$	
s^0	$5k$	

$\rightarrow -2k+6>0 \Rightarrow k<3$

$\rightarrow 5k>0$

$\therefore \boxed{3>k>0}$

(d) $X=\dfrac{1}{s^2}$

$$e(\infty)=\lim_{s\to 0} sE=\lim s\frac{1}{s^2}\left(\frac{1}{1+GH}\right)=\frac{1}{\frac{k5}{(1)(2)}}=\frac{1}{2}\Rightarrow \boxed{k=\frac{4}{5}}$$

consistent with $0<k<3$