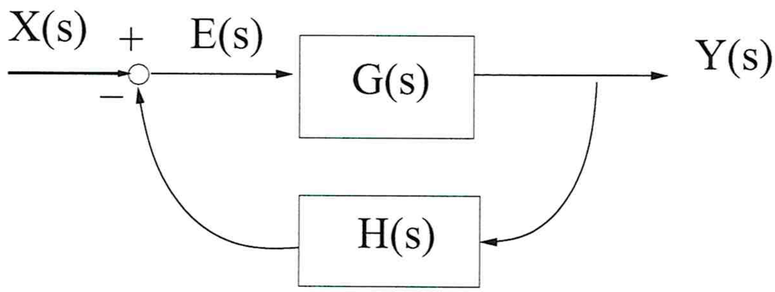
University of Massachusetts Lowell
Department of Electrical and Computer Engineering
16.413 Linear Feedback (4)

1.Given the negative feedback system where the open loop gain is

$$G(s) = \frac{s+b}{s(s+a)}$$



The feedback gain function $H(s)$ is equal to the constant K. The error $E(s) = X(s) - Y(s)H(s)$.

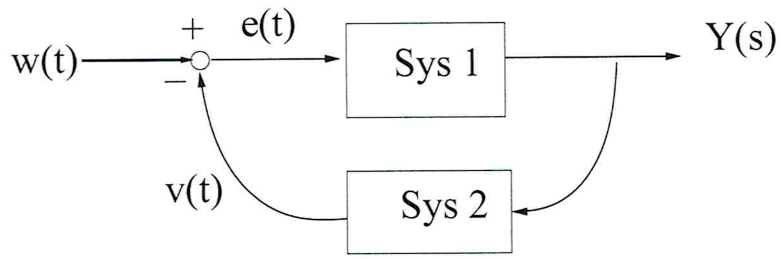
a. Determine the closed-loop transfer function $Y(s)/X(s)$

b. Determine the error transfer function $E(s)/X(s)$.

c. For $a > 0$, $b = 4a$ and the input $x(t) = (3+t)u(t)$ determine the steady state error $\lim_{t\to\infty} e(t)$.

d. Consider the case where the open-loop system is unstable and minimum-phase. In such a case $a < 0$ and $b > 0$. Determine the conditions on the gain of the controller for the closed-loop system to be stable.

2. Consider the negative feedback system



where each subsystem is governed by the equations: System 1:

$$\dot{x}_1 = -a x_1 + b\ e$$
$$y = c\ x_1$$

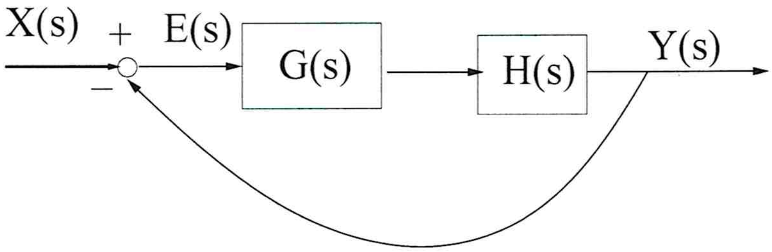
System 2:

$$\dot{x}_2 = -k x_2 + l\ e$$
$$v = x_2 + m\ y$$

a. Determine the transfer functions $Y/E, V/Y$ and Y/W.

b. State the conditions for stability of the closed-loop system in terms of a, b, c, k, l, m

3. Consider the negative feedback system



where

$$G(s) = \frac{1}{s(s+1)(s+2)}$$
$$H(s) = K\,(s+5)$$

a. Determine the closed-loop transfer function $Y(s)/X(s)$.

b. Determine the characteristic equation.

c. Define the conditions of K for stability.

d. The system error is defined as $e(t) = x(t) - y(t)$. Given the input $x(t) = tu(t)$ determine the constant K such that the steady state error is equal to 1/2. Is your result consistent with the stability condition obtained in part (c).