

$$(a) \quad \lambda_1 = -2 \quad \lambda_2 = -1 \quad \lambda_3 = -4$$

$$\underline{e}_1 = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} \quad \underline{e}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \quad \underline{e}_3 = \begin{bmatrix} 1 \\ -4 \\ 16 \end{bmatrix}$$

$$(b) \quad \Phi = \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ -2 & -1 & -4 \\ 4 & 1 & 16 \end{bmatrix}}_E \underbrace{\begin{bmatrix} e^{-2t} & 0 & 0 \\ 0 & e^{-t} & 0 \\ 0 & 0 & e^{-4t} \end{bmatrix}}_{e^{At}} \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ -2 & -1 & -4 \\ 4 & 1 & 16 \end{bmatrix}^{-1}}_{E^{-1}}$$

$$\Phi = \begin{bmatrix} \frac{8}{3}e^{-t} - 2e^{-2t} + \frac{e^{-4t}}{3} & 2e^{-t} - \frac{5}{2}e^{-2t} + \frac{e^{-4t}}{2} & \frac{e^{-t}}{3} - \frac{e^{-2t}}{2} + \frac{e^{-4t}}{6} \\ -\frac{8}{3}e^{-t} + 4e^{-2t} - \frac{4e^{-4t}}{3} & -2e^{-t} + 5e^{-2t} - 2e^{-4t} & -\frac{e^{-t}}{3} + e^{-2t} - \frac{2}{3}e^{-4t} \\ \frac{8}{3}e^{-t} - 8e^{-2t} + \frac{16}{3}e^{-4t} & 2e^{-t} - 10e^{-2t} + 8e^{-4t} & \frac{e^{-t}}{3} - 2e^{-2t} + \frac{8}{3}e^{-4t} \end{bmatrix}$$

$$(c) \quad E^{-1}B = \begin{bmatrix} -3 & -\frac{11}{2} \\ \frac{14}{3} & \frac{23}{3} \\ \frac{1}{3} & \frac{5}{6} \end{bmatrix} \quad \text{controllierbar}$$

$$(d) \quad CE = \begin{bmatrix} 14 & 3 & 60 \\ 4 & 1 & 16 \\ -10 & -4 & -28 \end{bmatrix} \quad \text{observierbar}$$

$$2. \quad A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad P = I$$

$$R = \begin{bmatrix} r_{11} & r_{12} \\ r_{12} & r_{22} \end{bmatrix}$$

AREqn

$$A^T R + R A - \overbrace{R B P^{-1} B^T} + Q = 0$$

$$\begin{bmatrix} -r_{12}^2 - 4r_{12} + 1 & \textcircled{3} -r_{12} r_{22} - 2r_{22} - 3r_{12} + r_{11} \\ -r_{12} r_{22} - 2r_{22} - 3r_{12} + r_{11} & \textcircled{2} -r_{22}^2 - 6r_{22} + 2r_{12} + 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\textcircled{1} \quad r_{12} = \sqrt{5} - 2 = 0.236$$

$$\textcircled{2} \quad r_{22} = \sqrt{2} \sqrt{\sqrt{5} + 3} - 3 = 0.236$$

$$\textcircled{3} \quad r_{11} = \sqrt{2} \sqrt{5} \sqrt{\sqrt{5} + 3} - 6 = 1.23$$

$$(b) \quad u^0 = -P^{-1} B^T R \underline{x} = -1 \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T \begin{bmatrix} 1.23 & 0.236 \\ 0.236 & 0.236 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \end{bmatrix} \begin{bmatrix} 1.23 & 0.236 \\ 0.236 & 0.236 \end{bmatrix} \underline{x} = \begin{bmatrix} -0.236 & -0.236 \end{bmatrix} \underline{x}$$

$$\underline{u}^0 = - (0.236x_1 + 0.236x_2)$$

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$$(a) H = x^2 + 2u^2 + \lambda_1(6x + u)$$

$$(b) \frac{\partial H}{\partial u} = 0 = 4u + \lambda_1 \Rightarrow \boxed{u^0 = -\frac{\lambda_1}{4}}$$

$$(c) H^0 = x^2 + \frac{2}{16} \lambda_1^2 + \lambda_1 \left(6x - \frac{\lambda_1}{4} \right)$$

$$\begin{cases} -\dot{\lambda}_1 = 2x + \lambda_1 \cdot 6 \\ \dot{x} = 6x - \lambda_1/4 \end{cases} \Rightarrow \begin{bmatrix} \dot{x} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} 6 & -\frac{1}{4} \\ -2 & -6 \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix}$$

(d)

$$z = e^{-6t} \cdot 1 + 0.007 [e^{-6t} - e^{6t}] \lambda(0)$$

$$\lambda(0) = \frac{z - e^{-6t}}{0.007(e^{-6t} - e^{6t})} \sim \frac{1}{0.007}$$

$$u^0 = -\frac{\lambda}{4} = \left(-\frac{1}{4} \right) \left[(e^{-6t} - e^{6t}) + \frac{e^{-6t}}{0.007} \right]$$