

$$1. \quad (a) \quad \frac{Y}{X} = \frac{\frac{4s+k}{s^2}}{1 + \frac{4s+k}{s^2(s+2)}} = \frac{(4s+k)(s+2)}{s^3 + 2s^2 + 4s + k}$$

$$(b) \quad s^3 + 2s^2 + 4s + k$$

$$(c) \quad \begin{array}{l} s^3 \\ s^2 \\ s^1 \\ s^0 \end{array} \left| \begin{array}{cc} 1 & 4 \\ 2 & k \\ \frac{8-k}{2} & \\ k & \end{array} \right.$$

$$\left. \begin{array}{l} \Rightarrow k < 8 \\ \Rightarrow k > 0 \end{array} \right\} \begin{array}{l} \text{stable} \\ 0 < k < 8 \end{array}$$

(d)

$$\left. \begin{array}{l} k=8 \\ P(s) = 2s^2 + k \end{array} \right\} 2s^2 + 8 = 0 \quad \boxed{s = \pm j2}$$

$$\omega = 2\pi f \text{ rad/sec}$$

$$\boxed{f = \frac{1}{\pi} \text{ Hz}}$$

2



$$h(t) = 5e^{-t} \cos(2t) u(t)$$

(a)

$$H(s) = \frac{A(s)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\zeta\omega_n = 1$$

$$\omega_d = 2 = \omega_n \sqrt{1 - \zeta^2} \Rightarrow 4 = \omega_n^2 (1 - \zeta^2) \Rightarrow 4 = \omega_n^2 \underbrace{(1 - \zeta^2)}_1$$

$$\boxed{\omega_n^2 = 5}$$

$$\therefore \zeta = \frac{1}{\sqrt{5}}$$

$$\omega_n = \sqrt{5}$$

$$(b) \quad H(s) = \int_0^{\infty} \frac{5}{2} \left[e^{-t} e^{j2t} + e^{-t} e^{-j2t} \right] e^{-st} dt$$

$$= \frac{5}{2} \left[\frac{1}{s+1-j2} + \frac{1}{s+1+j2} \right] \quad \text{Re}(s) > -1$$

$$= \frac{5}{2} \left[\frac{2(s+1)}{s^2 + 2s + 5} \right]$$

(c) $x(t) = u(t)$

$$y(t) = \int_0^t h(\hat{y}) d\hat{y} = \frac{5}{2} e^{-t} \int_0^t e^{-\hat{y}} e^{j2\hat{y}} d\hat{y}$$

$$y(t) = \left\{ 1 - e^{-t} \left\{ \cos(2t) - \sin(2t) \right\} \right\}$$

3

$$(a) GH+1=0 \Rightarrow (s+1)(s+2)(s+3)(s+4) + k(s-2) = 0$$

$$s^4 + 10s^3 + 35s^2 + (50+k)s + (24-2k) = 0$$

$$\begin{array}{r} s^4 \\ s^3 \end{array} \quad \begin{array}{l} 1 \\ 10 \end{array} \quad \begin{array}{l} 35 \\ (50+k) \end{array} \quad \begin{array}{l} 24-2k \end{array}$$

$$\begin{array}{r} s^2 \end{array} \quad \frac{350 - (50+k)}{10} \quad 24-2k \quad \Rightarrow \quad \begin{array}{l} 300-k > 0 \\ \boxed{300 > k} \end{array}$$

$$\frac{\frac{300-k}{10}}{10}$$

$$s^1 \quad \frac{(300-k)(50+k) - 10(24-2k)}{10} \quad \Rightarrow \quad (300-k)(50+k) - 100(24-2k) > 0$$

$$\frac{\frac{300-k}{10}}{10}$$

$$s^0 \quad 24-2k \quad \Rightarrow \quad \begin{array}{l} 24-2k > 0 \\ \boxed{12 > k} \end{array}$$

$$-k^2 + 450k + 12600 > 0$$

$$k^2 - 450k - 12600 < 0$$

$$(k-225)^2 - 225^2 - 12600 < 0$$

$$(k-225)^2 < 38025 \Rightarrow |k-225| < 195$$

$$k < 225 + \sqrt{38025} \quad 195$$

$$\boxed{30 < k < 420}$$

$$12 > k > -$$

$$(k-225)^2 < 225^2 + 12,600$$

$$-\sqrt{225^2 + 12,600} < k - 225 < \sqrt{225^2 + 12,600}$$

$$225 - \sqrt{225^2 + 12,600} < k < 225 + \sqrt{225^2 + 12,600}$$

$$\boxed{-26.4 < k < 476.4}$$

stability

$$(-26.4 < k < 476.4) \cap (k < 300) \cap (k < 12)$$

$$\boxed{-26.4 < k < 12}$$