University of Massachusetts Lowell
Department of Electrical and Computer Engineering
16.413 Linear Feedback

Problem set 6

1. Consider the system

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix} \underline{x} + \begin{bmatrix} 3a & 3b \\ 3 & 3 \end{bmatrix} \underline{u}$$

$$\underline{y} = \begin{bmatrix} \alpha & 1 \\ 2 & \beta \end{bmatrix} \underline{x}$$

a. Find the eigenvalues and eigenvectors of the system.

b. Find the state transition matrix.

c. Determine when the system is controllable.

d. Determine when the system is observable.

2. Consider the transfer function for a causal system

$$\frac{Y}{U} = \frac{10(s+4)}{s^3 + 9s^2 + 23s + 15}$$

where U and Y are the Laplace transform of the input and output respectively. Using phase variables representation of the system matrices (A, B, C) where $\dot{\underline{x}} = A\underline{x} + B\underline{u}$ and $\underline{y} = C\underline{x}$.

a. Determine the state and output equations in terms of the state vector $\underline{x}$, input u and output y.

b. Find the state transition matrix.

c. If $u(t) = \delta(t)$ determine $y(t)$ using the STM.

d. If $u(t)$ is equal to the unit step function what is $y(t)$. Explain your answer.

3. Consider the system

$$\dot{\underline{x}} = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \underline{x}$$

a. Find the STM and general solution for the arbitary initial condition $\underline{x}(0)$

b. Given the initial condition $x_3(0) = 0$ and terminal conditions $x_1(1) = 1$ and $x_2(1) = 2$ determine the required initial conditions $x_1(0)$ and $x_2(0)$.