16.413 Problem Set 6

1. Obtain the response \( y(t) \) of the unity negative feedback system to the input \( x(t) = \frac{t^2}{2} u(t) \). The open-loop gain is

\[
G(s) = \frac{1}{s(s + 2)}
\]

Determine the \( e(t) = y(t) - x(t) \)

2. Given the unity feedback system

\[
G(s) = \frac{K(s + 1)}{s^2} \quad H(s) = 1
\]

   a. Obtain the unit impulse response for \( K = 1 \).
   b. The unit-step response for \( K = 1 \).
   c. Plot the root loci as a function of \( K \) for the closed-loop system.

3. Given the unity feedback system

\[
G(s) = \frac{2K(s + 1)}{s^2(s + 5)(s + 2)} \quad H(s) = 1
\]

   a. Plot the root locus
   b. Determine the range of \( K \) where the closed-loop system is stable.

4. Given the unity feedback system

\[
G(s) = K \frac{(T_1 s + 1)}{s(T_2 + 1)(s + 1)} \quad H(s) = 1
\]

   a. Determine the values of \( K \), \( T_1 \) and \( T_2 \) such that the dominant closed-loop poles have a damping ratio \( \zeta = 0.5 \) and undamped natural frequency \( \omega_n = 3 \text{ rad/sec} \).