ECE4130/16.413 Problem Set #3

1. Consider the block diagram below.

(a) Evaluate the signal flow graph
(b) Determine the transfer function $Y(s)/X(s)$. 
2. Given the signal flow graph below determine the transfer matrix $A$ where $A_{ij} = Y_i/X_j$.

\[
\begin{bmatrix}
    Y_1 \\
    Y_2
\end{bmatrix} =
\begin{bmatrix}
    A_{11} & A_{12} \\
    A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
    X_1 \\
    X_2
\end{bmatrix}
\]

Note that $A_{ij} = Y_i/X_j$ given that all other inputs are equal to zero.
3. Given the system equations

\[
\begin{align*}
\frac{d^2 x_1}{dt^2} &= x_1 - 6x_2 \\
\frac{dx_2}{dt} &= 2x_1 + 4x_2 - u
\end{align*}
\]

a. Draw a signal-flow graph representation of the system using only integrators and amplifiers where \( U(s) \) is the input and \( X_1(s) \) is the output. You may assume zero initial conditions.

b. Find the transfer function \( X_1(s)/U(s) \) using Mason’s Gain formula. Check your result using an algebraic approach.
4. For the system below

\[ G(s) \]

- \[ X \]
- \[ + \]
- \[ G(s) \]
- \[ + \]
- \[ 2/s \]
- \[ Y \]

a. Determine the transfer function \( Y/X \)

b. If the error is defined as \( e(t) = x(t) - y(t) \) determine a \( G(s) \) such that \( e(\infty) = 1/2 \) when \( x(t) = t^2 u(t) \).
5. The steady-state error $e = x - y$ where $x$ is the input and $y$ is the output. Find the steady state error for the input is equal to $tu(t)$ for a unity negative feedback system with the open-loop transfer function

$$H(s)G(s) = \frac{100(s + 1)(s + 2)}{s(s + 3)(s + 10)}$$
6. The error in a unity negative feedback system is the error \( e(t) = x - y \) where \( x \) is the input and \( y \) is the output. The open loop transfer function is

\[
H(s)G(s) = H(s) \frac{500}{s + 50}
\]

a. For \( H(s) = 1 \) determine the steady state error for \( x = 5u(t) \)

b. Determine \( H(s) \) such that \( e(\infty) = 0.10 \) for \( x = 5t^2 u(t) \).
7. A unity feedback system with the open loop response

\[ G(s) = \frac{(s + \alpha)}{s(s + \beta)} \]

You are to design the system to meet the requirements: the steady state error for a unit ramp input equals 1/10; the closed-loop poles are located at \(-1 \pm j\). Find the value of \(H(s)\), \(\alpha\), and \(\beta\) required.