Test 3

1(a) $\lambda_1 = -9\,;\ \underline{e}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ $\qquad \lambda_2 = -6\,;\ \underline{e}_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

(b) $\underline{\Phi} = E e^{\Lambda t} \underline{E}^{-1} = \frac{1}{3} \begin{bmatrix} 4e^{-6t} - e^{-9t} & 2(e^{-6t} - e^{-9t}) \\ 2(e^{-9t} - e^{-6t}) & 4e^{-9t} - e^{-6t} \end{bmatrix}$

$E = \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix} \quad E^{-1} = \begin{bmatrix} -1 & -2 \\ 2 & 1 \end{bmatrix} \frac{1}{3} \quad e^{\Lambda t} = \begin{bmatrix} e^{-9t} & 0 \\ 0 & e^{-6t} \end{bmatrix}$

(c) $E^{-1} B = \frac{1}{3} \begin{bmatrix} -1 & -2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix} \leftarrow$ not cont.

(d) $C \underline{E} = \begin{bmatrix} 2 & 1 \\ -6 & -3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ 0 & -9 \end{bmatrix}$

$\uparrow$ not obs

(2)

$$\dot{x} = -x + u$$
$$V = \int_0^1 \frac{u^2}{2} dt$$

(a) $H = \frac{u^2}{2} + \lambda[-x+u]$

(b) $\frac{\partial H}{\partial u} = 0 \qquad u + \lambda = 0 \Rightarrow u^o = -\lambda \Big\} H_o = \frac{\lambda^2}{2} + \lambda[-x-\lambda]$

$$\left.\begin{aligned}\dot{x} &= -x-\lambda \\ \dot{\lambda} &= -\frac{\partial H_o}{\partial x} = \lambda\end{aligned}\right\} \begin{bmatrix}\dot{x} \\ \dot{\lambda}\end{bmatrix} = \begin{bmatrix}-1 & -1 \\ 0 & 1\end{bmatrix}\begin{bmatrix}x \\ \lambda\end{bmatrix}$$

$$E = \begin{bmatrix}1 & 1 \\ 0 & -2\end{bmatrix} \quad E^{-1} = \begin{bmatrix}1 & \frac{1}{2} \\ 0 & -\frac{1}{2}\end{bmatrix} \quad e^{\Lambda t} = \begin{bmatrix}e^{-t} & 0 \\ 0 & e^{t}\end{bmatrix}$$

(c) $$\Phi = E e^{\Lambda t} E^{-1} = \begin{bmatrix}e^{-t} & \frac{1}{2}(e^{-t}-e^{t}) \\ 0 & e^{t}\end{bmatrix}$$

$$\begin{matrix}x(t) \\ \lambda(t)\end{matrix} = \Phi(t)\begin{bmatrix}x(0) \\ \lambda(0)\end{bmatrix}$$

$x(0) = 1 \quad x(1) = 3$

$$\begin{bmatrix}3 \\ \lambda(1)\end{bmatrix} = \begin{bmatrix}e^{1} & \frac{1}{2}(e^{-1}-e^{1}) \\ 0 & e^{1}\end{bmatrix}\begin{bmatrix}1 \\ \lambda(0)\end{bmatrix}$$

$$\therefore 3 = e^{1} + \frac{1}{2}(e^{-1}-e^{1})\lambda(0)$$

$$\boxed{\lambda(0) = \frac{3-e^{1}}{\frac{1}{2}(e^{-1}-e^{1})}}$$

$$\boxed{u_o(t) = -\lambda(t) = -\lambda(0)e^{t}}$$

3

(a) $\dot{x} = -12x + u$

$V = \int_0^\infty 9x^2 + u^2 \, dt$

$A = -12 \qquad Q = 9$
$B = 1 \qquad P = 1$

$\left.\begin{matrix} Q = 9 \\ P = 1 \end{matrix}\right\} \quad Q - RB^TP^{-1}BR + RA + A^TR = 0$

scalar
$A^T = A$
$P^{-1} = \frac{1}{P}$

$9 - R\,1 \cdot 1 \cdot 1\,R - R\,12 - 12R = 0$

$-R^2 - 24R + 9 = 0 \Rightarrow R = \begin{cases} -3\sqrt{17} - 12 \\ \boxed{3\sqrt{17} - 12} \end{cases}$

$u^o = -P^{-1}BRx$

$$\boxed{u^o = -(1)(1)[3\sqrt{17} - 12]\,x}$$