16.413 Linear Feedback (7)

1.Using the HJ approach determine the input that minimize

$$V = \int_0^\infty x_1^2 + u^2 dt$$

subject to the constraints

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = u$$

a. Determine the A, B, Q and P matrices.

b. Determine the governing algebraic Riccati equation.

c. Find the optimal feedback gains and the resulting input u.

2. Consider the system

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \underline{x} + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \underline{u}$$
$$V = \int_0^\infty \left[x_1^2 + u_1^2 + u_2^2 \right] dt$$

a. Determine the governing Riccati equation.

3. For the second-order system

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \underline{u}$$
$$V = \int_0^1 \left[x_1^2 + u^2 \right] dt$$

a. Find the state function of Pontryagin

b. For $\underline{x}(0) = (0, 0)^T$ and $\underline{x}(1) = (1, 0)^T$ find the optimal input.