16.413 Linear Feedback (6)

1. Consider the linear system governed by the state equations:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} u \qquad y = [1\ 0\ 1] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

a. Determine the eigenvalues and eigenvectors of the system

b. Determine the state-transition matrix

c. Find the response $y(t)$ given $u(t) = \delta(t)$.

d. Determine $Y(s)/U(s)$.

2. Consider the dual input system

$$\dot{x}_1 = -6x_2 + u_1$$
$$\dot{x}_2 = \frac{x_1}{5} + \frac{u_2}{5}$$

a. Determine the elements of the transfer function matrix $A_{ij} = X_i/U_j$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

3. Consider the system

$$\dot{\underline{x}} = A\underline{x} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \underline{u} \qquad \underline{y} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \underline{x}$$

Given the matrix A has the eigenvalue/vector pairs $\lambda_1 = -3$; $\underline{e}_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ and $\lambda_2 = -1$; $\underline{e}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

a. Determine the conditions for the system to be *uncontrollable*.

b. Determine the conditions for the system to be *unobservable*.

4.Consider the system

$$\frac{dx}{dt} = x + u$$

where the performance of the system is given by $V = \int_0^1 u^2\,dt$.

a. Find the state function of Pontryagin H.

b. Using the state function determine the optimal input u^0.

c. Determine the equations governing λ and x.

d. Determine λ and x for $x(0) = 0$ and $x(1) = 1$.