

1.

(a) $H = u^2 + \lambda(12x + 4u)$

(b) $\frac{\partial H}{\partial u} = 0 \quad 2u + 4\lambda = 0 \Rightarrow \boxed{u = -2\lambda}$

$H^0 = 4\lambda^2 + \lambda[12x - 8\lambda]$

(c)
$$\begin{cases} \dot{x} = 12x - 8\lambda \\ -\dot{\lambda} = 12\lambda \end{cases}$$

$$\begin{bmatrix} \dot{x} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} 12 & -8 \\ 0 & -12 \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix}$$

Find $\Phi \quad E = \begin{bmatrix} 1 & 1 \\ 3 & 0 \end{bmatrix}; E^{-1} = \begin{bmatrix} 0 & \frac{1}{3} \\ 1 & -1/3 \end{bmatrix}; e^{\Delta t} = \begin{bmatrix} e^{-12t} & 0 \\ 0 & e^{12t} \end{bmatrix}$

$$\Phi = E e^{\Delta t} E^{-1} = \begin{cases} e^{12t} & \frac{1}{3}[e^{-12t} - e^{12t}] \\ 0 & e^{-12t} \end{cases}$$

(d) $x(0) = 1; x(1) = 2$

$$\begin{bmatrix} x(t) \\ \lambda(t) \end{bmatrix} = \begin{cases} e^{12t} & \frac{1}{3}[e^{-12t} - e^{12t}] \\ 0 & e^{-12t} \end{cases} \begin{bmatrix} 1 \\ \lambda(0) \end{bmatrix}$$

$$2 = e^{12(1)} + \frac{1}{3}[e^{-12(1)} - e^{12(1)}] \lambda(0)$$

$$\lambda(0) = \frac{(2 - e^{12})3}{e^{-12} - e^{12}}$$

$$\begin{cases} x(t) = e^{12t} + \frac{1}{3}[e^{-12t} - e^{12t}] \left\{ \frac{3(2 - e^{12})}{e^{-12} - e^{12}} \right\} \\ \lambda(t) = e^{-12t} \left\{ \frac{3(2 - e^{12})}{e^{-12} - e^{12}} \right\} \end{cases}$$

2.

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad BB^T = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad P = 1$$

$$R = \begin{bmatrix} r_{11} & r_{12} \\ r_{12} & r_{22} \end{bmatrix}$$

$$-R \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} R + Q + A^T R + R A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -r_{12}^2 - 2r_{12} + 1 & -r_{12}r_{22} - r_{22} - r_{12} + r_{11} \\ -r_{12}r_{22} - r_{22} - r_{12} + r_{11} & -r_{22}^2 - 2r_{22} + 2r_{12} + 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$-r_{12}^2 - 2r_{12} + 1 = 0 \Rightarrow r_{12} = 0.41$$

$$1 + 2r_{12} - 2r_{22} - r_{22}^2 = 0 \Rightarrow r_{22} = 0.68$$

$$r_{11} - r_{12} - r_{22} - r_{12}r_{22} = 0 \Rightarrow r_{11} = \frac{1711}{1250} = 1.37$$

$$K = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1.37 & 0.41 \\ 0.41 & 0.68 \end{bmatrix} = \begin{bmatrix} 0.41 & 0.68 \end{bmatrix}$$

$$x = - \begin{bmatrix} 0.41 & 0.68 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$