Test3

1. $$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix} \underline{x} + \begin{bmatrix} 3a & 3b \\ 3 & 3 \end{bmatrix} \underline{u}$$

$$\underline{y} = \begin{bmatrix} \alpha & 1 \\ 2 & \beta \end{bmatrix} \underline{x}$$

(a) $\underline{e}_1 = \begin{bmatrix} 1 \\ -4 \end{bmatrix} \quad \lambda_1 = -4 \qquad \underline{e}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \lambda_2 = -1$

$$E = \begin{bmatrix} 1 & 1 \\ -4 & -1 \end{bmatrix} \quad \Bigg| \quad E^{-1} = \begin{bmatrix} -1/3 & -1/3 \\ 4/3 & 1/3 \end{bmatrix} \quad \Bigg| \quad e^{\Lambda t} = \begin{bmatrix} e^{-4t} & 0 \\ 0 & e^{-t} \end{bmatrix}$$

(b) $\Phi = E e^{\Lambda t} E^{-1}$

$$\Phi = \frac{1}{3} \begin{bmatrix} 4e^{-t} - e^{-4t} & e^{-t} - e^{-4t} \\ 4e^{-4t} - 4e^{-t} & 4e^{-4t} - e^{-t} \end{bmatrix}$$

(c) $$E^{-1} B = \begin{bmatrix} -a-1 & -b-1 \\ 4a+1 & 4b+1 \end{bmatrix} \Longrightarrow \left. \begin{matrix} a = -1 \;\&\; b = -1 \\ a = b = -1/4 \end{matrix} \right\} \text{unc}$$

~~Controllable for all a & b~~

$\Downarrow$ $\Downarrow$

~~a+1~~

(d) ~~$CE = \begin{bmatrix} a & 1 \\ 2 & b \end{bmatrix} E$ obs for all a & b~~

$$CE = \begin{bmatrix} \alpha - 4 & \alpha - 1 \\ 2 - 4\beta & 2 - \beta \end{bmatrix}$$

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~~not cont~~ $\underbrace{\alpha = 4 \;\&\; \beta = \tfrac{1}{2} \qquad \alpha = 1 \;\&\; \beta = 2}_{\text{not obs}}$

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(a) $$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -4 & -4 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} \underline{u} \qquad \frac{Y}{U} = \frac{10(s+4)}{(s-1)(s+2)(s+3)}$$

$$y = \begin{bmatrix} 4 & 1 & 0 \end{bmatrix} \underline{x}$$

(b) $$\Phi = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix} \begin{bmatrix} e^{t} & 0 & 0 \\ 0 & e^{-2t} & 0 \\ 0 & 0 & e^{-3t} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix}^{-1}$$

$$\underline{x} = \Phi \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$y = \begin{bmatrix} 4 & 1 & 0 \end{bmatrix} \Phi \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} = \frac{25}{6}e^{t} - \frac{20}{3}e^{-2t} + \frac{5}{2}e^{-3t}$$

$$\underline{B}^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{5}{12} & \frac{1}{12} \\ 1 & -\frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$$

$$\Phi = \begin{bmatrix} \frac{e^{t} - e^{-3t}}{2} + e^{-2t} & \frac{5e^{t}}{12} - \frac{2}{3}e^{-2t} + \frac{e^{-3t}}{4} & \frac{e^{t}}{12} - \frac{e^{-2t}}{3} + \frac{e^{-3t}}{4} \\ \frac{e^{t}}{2} - 2e^{-2t} + \frac{3}{2}e^{-3t} & \frac{5}{12}e^{t} + \frac{4e^{-2t}}{3} - \frac{3}{4}e^{-3t} & \frac{e^{t}}{12} + \frac{2}{3}e^{-2t} - \frac{3}{4}e^{-3t} \\ \frac{e^{t}}{2} + 4e^{-2t} - \frac{9}{2}e^{-3t} & \frac{5}{12}e^{t} - \frac{8}{3}e^{-2t} + \frac{9}{4}e^{-3t} & \frac{e^{t}}{12} - \frac{4}{3}e^{-2t} + \frac{9}{4}e^{-3t} \end{bmatrix}$$