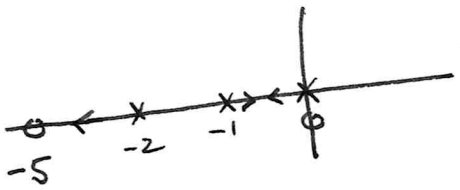
Test 2
16.413

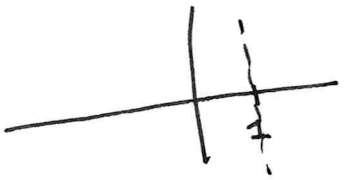
1.

$$GH = \frac{K(s+5)}{s(s+1)(s+2)}$$

(a)



(b) $\sigma = \frac{-2-1-0+5}{3-1} = \frac{2}{2} = 1$



$$\phi = \pm\frac{\pi}{2}$$

(c) $GH+1=0 \qquad s(s+1)(s+2)+K(s+5)=0$

$$s^3+3s^2+s(K+2)+5K=0$$

s^3	1	$(K+2)$
s^2	3	$5K$
s^1	$\frac{3(K+2)-5K}{3}$	$\Rightarrow -2K+6$
s^0	$5K$	

$K < \frac{6}{2} = 3$

$K > 0$

stable $\frac{6}{2} > K > 0$

marginal stable

$K = \frac{6}{2} = 3$

$3s^2 + \frac{5}{2} = 0$

$s = \pm j\sqrt{\frac{5}{6}}$

$\omega = \sqrt{\frac{5}{6}}$

$f = \frac{1}{2\pi}\sqrt{\frac{5}{6}}$

$3s^2 + \frac{5\cdot 6}{3} = 0$

$3s^2 + 10 = 0 \Rightarrow s = \pm j\sqrt{\frac{10}{3}}$

$3s^2 + 15 = 0$

$s^2 = -\frac{15}{3} = -5$

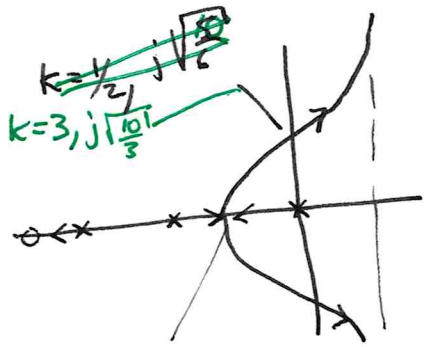
$s = \pm j\sqrt{5}$

(d) $$k = -\frac{s(s+1)(s+2)}{(s+5)}$$

$$\frac{dk}{ds} = 0 = -\frac{2s^3 + 18s^2 + 30s + 10}{(s+5)^2}$$

$s = -0.45$ $\quad$ $k = 0.84$

(e)



-0.45, $k = 0.84$

2.

(a) $GH+1=0$ $\quad \frac{ks+10}{s^2(s+4)}$

$$s^3+4s^2+sk+10=0$$

(b)

s^3	1	k
s^2	4	10
s^1	$\frac{4k-10}{4}$	
s^0	40	

$4k-10>0$

$$\boxed{k>\frac{10}{4}}$$

(c) $\boxed{k=\frac{10}{4}}$

$4s^2+40=0$

$$\boxed{s=j\sqrt{\frac{10}{4}}}$$

$$\left(s^2+\frac{10}{4}\right)\overline{)\,s^3+4s^2+s\frac{10}{4}+10\;\;\big|\;\;s+4}$$

$s^3 \quad s\frac{10}{4}$

$4s^2 \quad +10$

$4s^2 \quad +10$

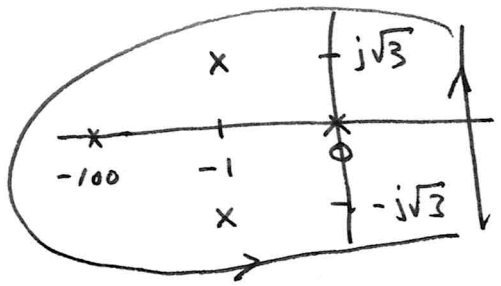
0

$$\boxed{s=\pm j\sqrt{\frac{10}{4}}}$$

$$\boxed{s=-4}$$

(3)

(a) $X(s) = \frac{1}{s}$



$t>0$

$t>0$

$$y(t) = \frac{1}{2\pi i}\int X(s)e^{st}\Big|_{(s+100)2\pi i} + \frac{1}{2\pi i}\int X(s)e^{st}\Big|_{(s+1+j\sqrt{3})2\pi i}$$

$s=-100$ $\qquad$ $s=-1-j\sqrt{3}$

$$+\frac{1}{2\pi i}\int e^{st}\Big|_{(s+1-j\sqrt{3})2\pi i}$$

$s=-1+j\sqrt{3}$

$$y(t) = 1 - \frac{e^{-100t}}{2451} - e^{-t}\left\{\frac{2450}{2451}\cos(\sqrt{3}t) + \frac{850}{817}\frac{\sin\sqrt{3}t}{\sqrt{3}}\right\}$$

$$y \sim 1 - e^{-t}\{\ \} \Rightarrow \sqrt{3}t = \pi \Rightarrow t = \frac{\pi}{\sqrt{3}}$$

$$y\left(\frac{\pi}{\sqrt{3}}\right) = 1 + e^{-\frac{\pi}{\sqrt{3}}}$$

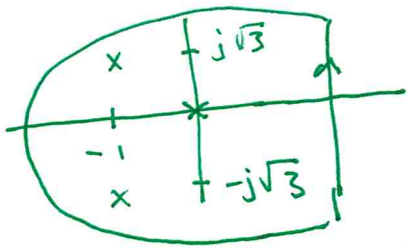
$$M_p = e^{-\pi/\sqrt{3}}$$

(3)

$X = \frac{1}{s}$

(b) $Y = \frac{400}{s[s+100][s^2+2s+4]} \sim \frac{400}{s[s^2+2s+4]100}$

Look retain the dom. pole



FVT $y(\infty) = 1$ ✓

$Y(s) = \frac{4}{s[s^2+2s+4]}$

$2\zeta\omega_n = 2$, $\omega_n^2 = 4$ $\Rightarrow$ $2\zeta 2 = 4$ $\boxed{\zeta = \frac{1}{2}, \omega_n = 2}$

$M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} = e^{\frac{-\frac{1}{2}\pi}{\sqrt{1-1/4}}} = e^{\frac{-\frac{1}{2}\pi}{\sqrt{3/4}}} = e^{-\frac{\pi}{\sqrt{3}}}$