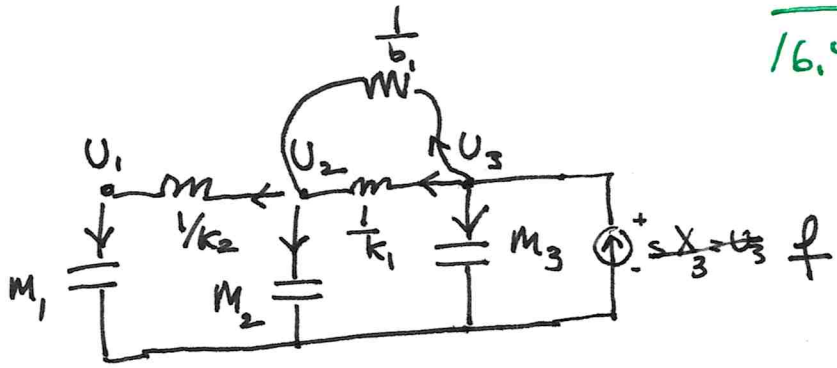
Test 1

16.413

1.



(b) U_3: $f = \dfrac{U_3}{\frac{1}{M_3 s}} + \dfrac{U_3 - U_2}{\frac{s}{K_1}} + \dfrac{U_3 - U_2}{\frac{1}{b_1}}$

U_2: $\dfrac{U_3 - U_2}{\frac{s}{K_1}} + \dfrac{U_3 - U_2}{\frac{1}{b_1}} = \dfrac{U_2 - U_1}{\frac{s}{K_2}} + \dfrac{U_2}{\frac{1}{M_2 s}}$

U_1: $\dfrac{U_2 - U_1}{\frac{s}{K_2}} = \dfrac{U_1}{\frac{1}{M_1 s}}$

(c) $f = \ddot{x}_3 M_3 + (x_3 - x_2) K_1 + (\dot{x}_3 - \dot{x}_2) b_1$

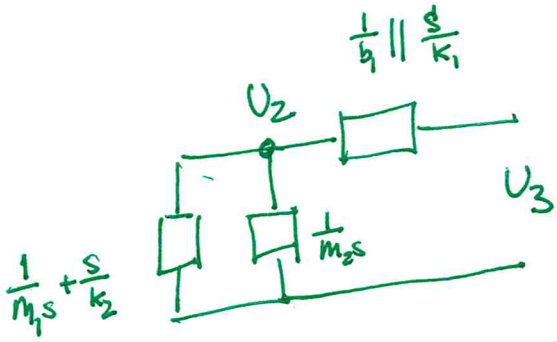
$(x_3 - x_2) K_1 + (\dot{x}_3 - \dot{x}_2) b_1 = (x_2 - x_1) K_2 + \ddot{x}_2 M_2$

$(x_2 - x_1) K_2 = \ddot{x}_1 M_1$

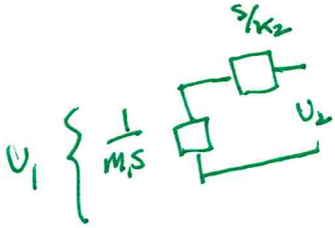
(d) $U_2 = U_1 \left(1 + \dfrac{M_1}{K_2} s^2 \right)$

$U_3 \left(\dfrac{K_1}{s} + b_1 \right) = U_2 \left[\dfrac{K_1}{s} + \dfrac{K_2}{s} + b_1 + M_2 s \right] - \left(\dfrac{K_2}{s} \right) U_1$

$$\frac{U_1}{U_3} = \frac{K_2 K_1 + s b_1 K_2}{\left[K_2 + M_1 s^2 \right] \left[K_1 + K_2 + b_1 s + M_2 s^2 \right] - K_2} = \frac{b_1 K_2 s + K_1 K_2}{M_1 M_2 s^4 + b_1 M_1 s^3 + \left[M_2 K_2 + M_1 (K_1 + K_2) \right] s^2 + K_2 b_1 s + K_1 K_2}$$



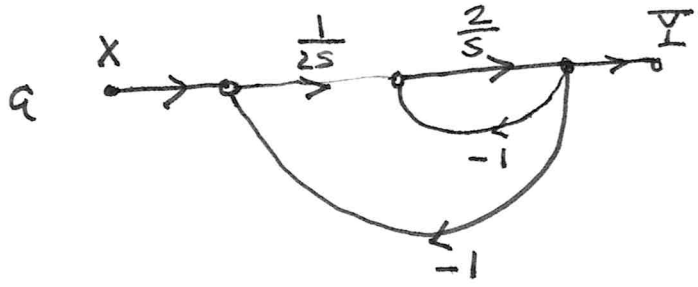
$$\frac{U_2}{U_3} = \frac{\left(\frac{1}{M_1 s} + \frac{s}{K_2}\right) \| \frac{1}{M_2 s}}{\left(\frac{1}{b_1} \| \frac{s}{K_1}\right) + \frac{1}{M_2 s} \| \left(\frac{1}{M_1 s} + \frac{s}{K_2}\right)}$$



$$\frac{U_1}{U_2} = \frac{\frac{1}{M_1 s}}{\frac{1}{M_1 s} + \frac{s}{K_2}}$$

$$\frac{U_1}{U_3} = \frac{U_1}{U_2} \frac{U_2}{U_3}$$

3.



(b) $P_1 = \frac{1}{2s} \cdot \frac{2}{s} = \frac{1}{s^2}$

Loops $L_1 = -\frac{2}{s}\frac{1}{2s} = -\frac{1}{s^2}$ $\quad \Delta = 1 - (L_1 + L_2)$

$L_2 = -\frac{2}{s}$ $\quad \Delta_1 = 1$

$$\frac{Y}{X} = \frac{P_1\Delta_1}{\Delta} = \frac{\frac{1}{s^2}}{1 + \left[\frac{1}{s^2} + \frac{2}{s}\right]} = \frac{1}{s^2 + 2s + 1}$$

2

(a) $E(s) = X - HY \Rightarrow \frac{E}{X} = 1 - H\frac{Y}{X}$

$\frac{Y}{X} = \frac{G}{1+GH}$

$\boxed{\frac{E}{X} = \frac{1}{1+GH}} \Rightarrow \frac{E}{X} = \frac{1}{1+\left[\frac{s+4}{s^2(s+15)}\right]H}$

(b) $X(s) = \frac{3}{s} + \frac{1}{s^2} = \frac{1+3s}{s^2}$

$e(\infty) = \uparrow_{s}^{0} sE = \uparrow_{s}^{0} \frac{1+3s}{s+\left[\frac{s+4}{s(s+15)}\right]H} = \uparrow_{s}^{0} \frac{1}{\frac{4}{15}\frac{1}{s}H} = \frac{1}{10}$

$\boxed{H = ks}$

$\frac{1}{\frac{4}{5}ks} = \frac{1}{10} \Rightarrow k \cdot \frac{4}{5} = 10$

$\boxed{k = \frac{150}{4}}$