University of Massachusetts Lowell
Department of Electrical and Computer Engineering

16.413 Linear Feedback

Problem set 3

1. The steady-state error in the velocity of a system is defined as

$$\underset{t\to\infty}{limit}\left(\frac{dx}{dt}-\frac{dy}{dt}\right)$$

where x is the input and y is the output. Find the steady state error in the velocity when the input is equal to $t^2u(t)$ for a unity feedback system with the open-loop transfer function

$$G(s)=\frac{100(s+1)(s+2)}{s^2(s+3)(s+10)}$$

2. The error in a unity feedback system is the error $e(t) = x - y$ where x is the input and y is the output. The open loop transfer function is

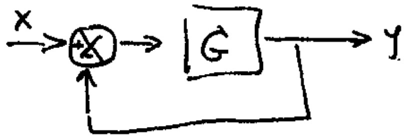
$$G(s)=\frac{5000}{s(s+75)}$$

(a) Determine the steady state error for $x = 5u(t)$
(b) Determine the steady state error for $x = 5t^2u(t)$.

3. A unity feedback system with the open loop response

$$G(s)=\frac{K(s+\alpha)}{s(s+\beta)}$$

You are to design the system to meet the requirements: the steady state position error for a unit ramp input equals $1/10$; the closed-loop poles are located at $-1 \pm j$. Find the values of K, α and β required.

① 

$$\frac{Y}{X} = \frac{G}{1+G}$$

$$E = s\left[X - Y\right] \Rightarrow \frac{E}{X} = s\left[1 - \frac{Y}{X}\right] = s\left[\frac{1}{1+G}\right]$$

$$X = \frac{2}{s^3}$$

$$E = \left(\frac{2}{s^3}\right)s\left[\frac{1}{1+G}\right] = \frac{2}{s^2}\left[\frac{1}{1+G}\right]$$

$$e(\infty) = \lim_{s \to 0} \frac{2}{s^2}\left[\frac{1}{1+G}\right] = \lim_{s \to 0} \frac{2}{sG} = 0$$

2

$$\frac{E}{X} = \frac{1}{1+G}$$

(a) $X = \frac{5}{s}$

$$E = \left(\frac{5}{s}\right)\left(\frac{1}{1+G}\right) \Rightarrow sE = \frac{5}{1+G}$$

$$e(\infty) = \lim_{s \to 0} sE = \lim_{s \to 0} \frac{5}{1+G} = \frac{5}{1+\lim_{s \to 0} G} = 0$$

(b) $X = \cancel{\frac{5}{s(t)}}\ \frac{5 \times 2}{s^2}$

$$sE = \frac{5s \times 2}{s^2[1+G]} \Rightarrow e(\infty) = \lim_{s \to 0} \frac{5 \times 2}{s[1+G]} = \frac{5 \times 2}{1+\frac{5000}{75}} = e(\infty$$

$$\cancel{\lim_{s \to 0} \frac{5}{5G}}$$

3

(a) $\dfrac{Y}{X} = \dfrac{G}{1+G}$

$E = X - Y \Rightarrow \dfrac{E}{X} = \dfrac{1}{1+G}$

(i) $x = tu(t)$ $\quad e(\infty) = \cancel{\ } \frac{1}{10}$

$e(\infty) = \lim_{s\to 0} sE = \lim_{s\to 0} s\frac{1}{s^2}\left[\frac{1}{1+G}\right] \Rightarrow \lim_{s\to 0} \frac{1}{sG} = \cancel{\lim_{s\to 0} \frac{k\alpha}{\ }}$

$= \lim_{s\to 0} \dfrac{1}{\dfrac{sk(s+\alpha)}{s(s+\beta)}} = \boxed{\dfrac{\beta}{k\alpha} = \dfrac{1}{10}}$

(ii) $1+G=0$ for $s = -1 \pm j$

$1 + \dfrac{k(s+\alpha)}{s(s+\beta)} = 0 \Rightarrow \boxed{s^2 + (k+\beta)s + \alpha k = 0}$

$\therefore (-1+j)^2 + (k+\beta)(-1+j) + \alpha k = 0 \rightarrow j[k+\beta-2] + [\alpha k - k - \beta] = 0$

or $(-1-j)^2 + (k+\beta)(-1-j) + \alpha k = 0$

$\boxed{\begin{aligned} k+\beta-2 &= 0 \\ \alpha k - k - \beta &= 0 \end{aligned}}$ *

then $\boxed{\begin{aligned} \alpha &= \frac{10}{9} \\ \beta &= 0.2 \\ k &= 1.8 \end{aligned}}$