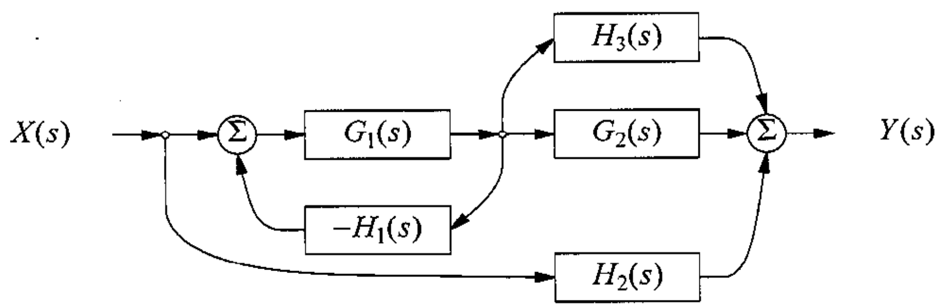
Department of Electrical and Computer Engineering
University of Massachusetts Lowell

16.413 Problem Set #2

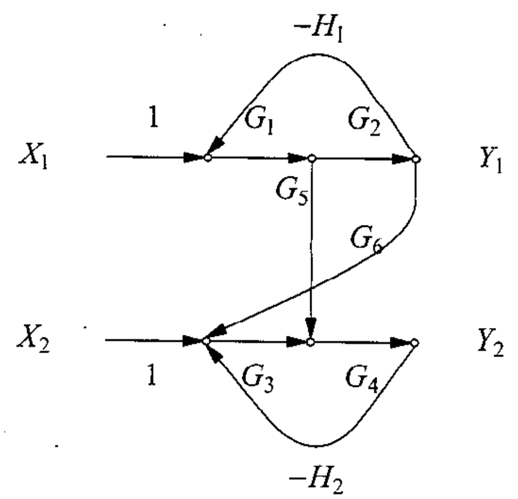
1. Consider the block diagram below.



(a) Evaluate the signal flow graph
(b) Determine the transfer function $Y(s)/X(s)$.

2. Given the signal flow graph below determine the transfer matrix A where $A_{ij} = Y_i/X_j$.

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$



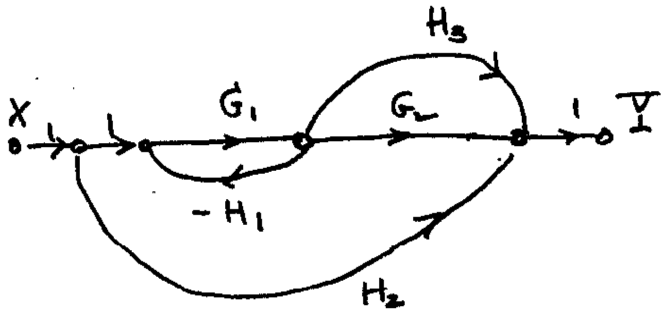
3. Verify that x_1, x_2 and x_3 can be determined using a signal flow graph where $u_1 = 2$ and $u_2 = 1$.

$$x_1 = -x_2 - 3x_3 + u_1$$
$$x_2 = 5x_1 - x_2 - x_3$$
$$x_3 = 4x_1 + x_2 - 5x_3 + u_2$$

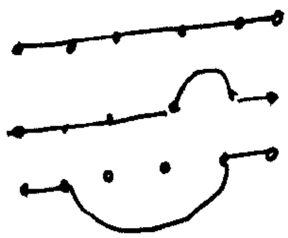
2

1\.

(a)



(b) Forward path:



$P_1 = G_1 G_2$

$P_2 = G_1 H_3$

$P_3 = H_2$

<u>Loop</u>

$L_1 = -G_1 H_1$

$\Delta = 1 - L_1$

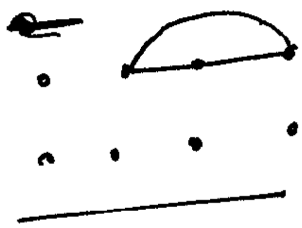
$\Delta_1 = 1$

$\Delta_2 = 1$

$\Delta_3 = 1 - L_1$

$$\frac{Y}{X} = \frac{P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3}{\Delta}$$

Loops



$L_1 = G_1 G_2 (-H_1)$

$L_2 = G_3 G_4 (-H_2)$

$$A_{11} = \left(\frac{Y_1}{X_1}\right)\Bigg|_{X_2=0} = \frac{P_1 \Delta_1}{\Delta}$$

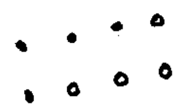
Forward path

$P_1 = G_1 G_2$

$\Delta_1 = 1 - L_2$

$$A_{12} = \left(\frac{Y_1}{X_2}\right)\Bigg|_{X_1=0} = 0$$

Forward path



$P_1 = 0$

$$A_{21} = \left(\frac{Y_2}{X_1}\right)\Bigg|_{X_2=0} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

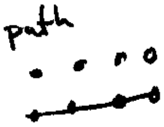
Forward path

$P_1 = G_1 G_5 G_4$; $\Delta_1 = 1$

$P_2 = G_1 G_2 G_6 G_3 G_4$; $\Delta_2 = 1$

$$A_{22} = \left(\frac{Y_2}{X_2}\right)\Bigg|_{X_1=0} = \frac{P_1 \Delta_1}{\Delta}$$

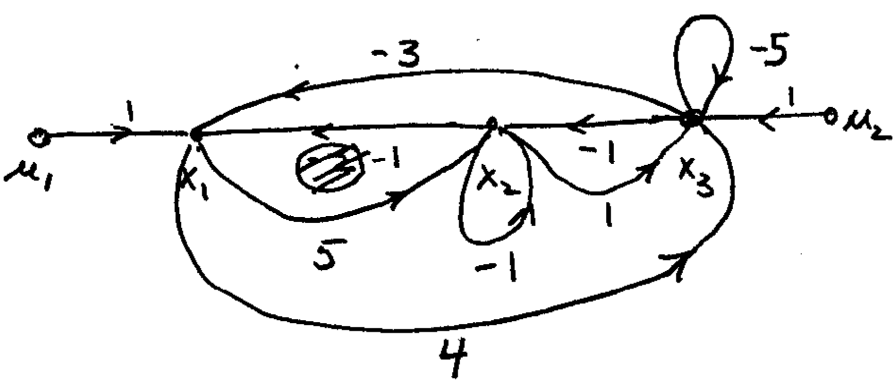
Forward path



$P_1 = G_3 G_4$

$\Delta_1 = 1 - L_1$

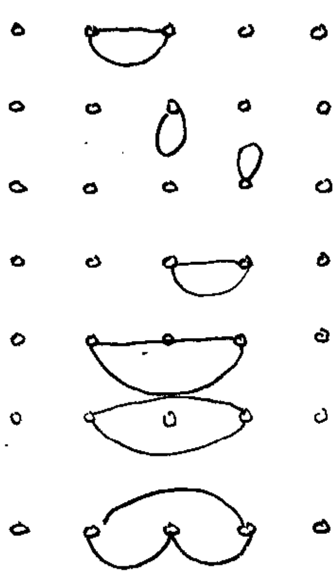
3



direct

$$\left.\begin{aligned} x_1 &= \frac{13}{78}u_1 - \frac{5}{78}u_2 \\ x_2 &= \frac{13}{39}u_1 - \frac{8}{39}u_2 \\ x_3 &= \frac{7}{78}u_2 + \frac{13}{78}u_1 \end{aligned}\right\} \Leftarrow \begin{bmatrix} 1 & 1 & 3 \\ -5 & 2 & 1 \\ -4 & -1 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} u_1 \\ 0 \\ u_2 \end{bmatrix}$$

Loops



$L_1 = (-1)(5) = -5$

$L_2 = -1$

$L_3 = -5$

$L_4 = (-1)(1) = -1$

$L_5 = (-1)(-1)(4) = 4$

$L_6 = (-3)(4) = -12$

$L_7 = (5)(1)(-3) = -15$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7) + [L_1 L_3 + L_2 L_3 + L_2 L_6]$$

$\boxed{\Delta = 78}$

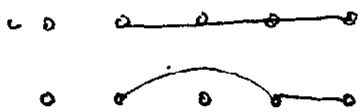
$\frac{X_1}{u_1}$

$P_1 = 1$

$\Delta_1 = 1 - [L_2 + L_3 + L_4] + [L_2 L_3 + \cancel{L_2}L_6] = 13$

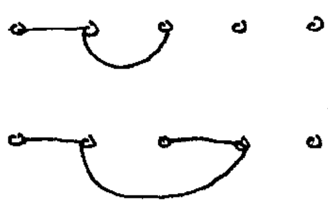
$\frac{P_1}{\Delta} \Delta_1 = \frac{13}{78}$

$\frac{X_1}{u_2}$



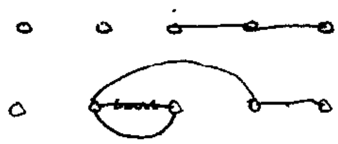
$\left.\begin{array}{l} P_1 = 1 \\ \Delta_1 = 1 \\ P_2 = -3 \\ \Delta_2 = 1 - L_2 = 2 \end{array}\right\} \quad \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{1-6}{78} = \frac{-5}{78}$

$\frac{X_2}{u_1}$



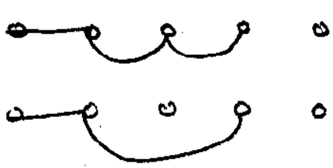
$\left.\begin{array}{l} P_1 = 5 \\ \Delta_1 = 1 - L_3 = 6 \\ P_2 = (4)(-1) = -4 \\ \Delta_2 = 1 \end{array}\right\} \quad \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{30-4}{78} = \frac{26}{78}$

$\frac{X_2}{u_2}$



$\left.\begin{array}{l} P_1 = (-1)(1) = -1 \\ \Delta_1 = 1 \\ P_2 = \cancel{(-1)(-3)(-1)}\ (1)(-3)(5) = -15 \\ \Delta_2 = 1 \end{array}\right\} \quad \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{-1-15}{78} = \frac{-16}{78}$

$\frac{X_3}{u_1}$



$\left.\begin{array}{l} P_1 = (1)(5)(1) = 5 \\ \Delta_1 = 1 \\ P_2 = (1)(4) = 4 \\ \Delta_2 = (1 - L_2) = 2 \end{array}\right\} \quad \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{5+8}{78} = \frac{13}{78}$

$\frac{X_3}{u_2}$

$\left.\begin{array}{l} P_1 = 1 \\ \Delta_1 = 1 - (L_1 + L_2) = 7 \end{array}\right\} \quad \frac{P_1\Delta_1}{\Delta} = \frac{7}{78}$

$X_1 = \frac{13}{78} u_1 - \frac{5}{78} u_2$

$X_2 = \frac{26}{78} u_1 - \frac{16}{78} u_2$

$X_3 = \frac{13}{78} u_1 + \frac{7}{78} u_2$