University of Massachusetts Lowell
Department of Electrical and Computer Engineering

16.413 Linear Feedback

Problem set 1

FIGURE 1

(A) (B)

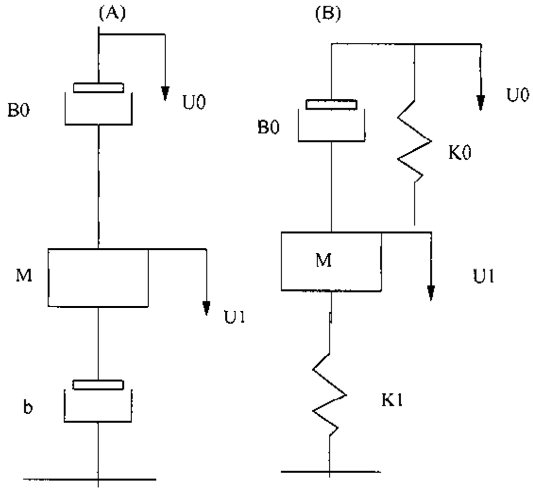


FIGURE 2

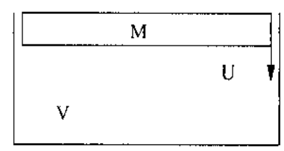
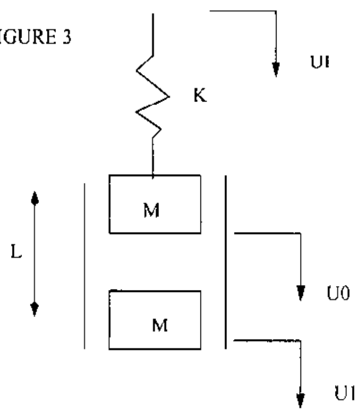


FIGURE 3



1.In Figure one the velocities are denoted by the symbols $U0$ and $U1$.

a. Using the mobility analogy where the velocity is the across variable and force is the through variable obtain the circuit model for each system.

b. Determine the transfer function $U1(s)/U0(s)$.

c. If $M = 1(kg)$, $K0 = K1 = 2(Nt/m)$, and $b = B0 = 3(Nt \cdot s/m)$ determine the response $u1(t)$ given that $u0(t) = \delta(t)$. Find the poles of the transfer function.

2. In Figure 2 you are given a Mass M situation in a box of volume V. The fluid (air) in the box has a sound speed $c = 345(m/s)$ and density $\rho = 1.18(kg/m^{3})$. The system is initial at rest and at equilibrium. You may also consider that its dimensions are small when compared to a wavelength for the frequencies of interest. If mass is hit with a hammer at what frequency will the mass vibrate? Please use a model to support your claim.

3. In Figure 3 you given 2 masses of mass M coupled by air enclosed in a tube. The cross sectional area of the tube is A and the length L. You consider the the acceleration due to gravity' is equal to zero.

a. Determine a circuit analog for the system using the mobility analogy.

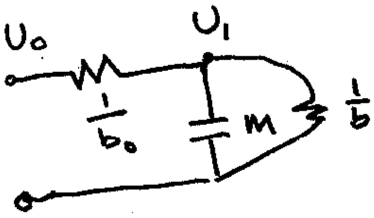
b. Determine transfer function $U1(s)/U0(s)$.

c. Please determine the force acting on the lower mass in terms of $U0(s)$.

d. Determine the ODEs governing the motion in the system.

1

(a)

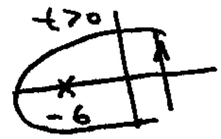


$$\frac{U_1}{U_0} = \frac{\frac{1}{Ms} \,\|\, \frac{1}{b}}{\frac{1}{b_0} + \frac{1}{Ms} \,\|\, \frac{1}{b}} = \frac{b_0}{sM + b + b_0}$$

$U_0(s) = 1$

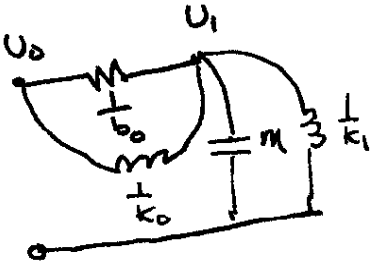
$$U_1 = \frac{3}{s+3+3} = \frac{3}{s+6}$$

$t>0$



$u_1(t) = 3e^{-6t}\mu(t)$

poles: $s = -6$



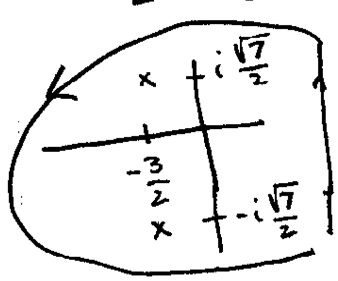
$$\frac{U_1}{U_0} = \frac{\frac{1}{Ms} \,\|\, \frac{s}{k_1}}{\frac{1}{b_0} \,\|\, \frac{s}{k_0} + \frac{1}{Ms} \,\|\, \frac{s}{k_1}} = \frac{b_0 s + k_0}{s^2 M + b_0 s + k_0 + k_1}$$

$U_0(s) = 1$

$$U_1 = \frac{3s+2}{s^2+3s+4} = \frac{3s+2}{\left(s+\frac{3}{2}+i\frac{\sqrt{7}}{2}\right)\left(s+\frac{3}{2}-i\frac{\sqrt{7}}{2}\right)}$$

poles $s = -\frac{3}{2} \pm i\frac{\sqrt{7}}{2}$

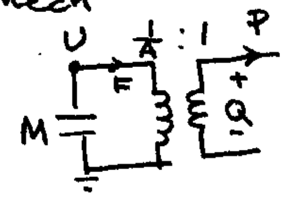
$t>0$

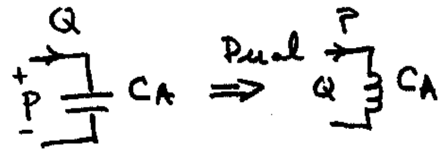


$\underline{t>0}$

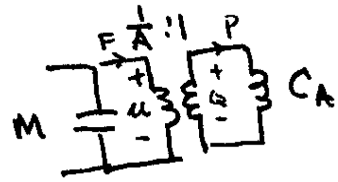
$$u_1(t) = e^{-\frac{3}{2}t}\left[3\cos\left(\frac{\sqrt{7}}{2}t\right) - \frac{5}{\sqrt{7}}\sin\left(\frac{\sqrt{7}}{2}t\right)\right]$$

Mech

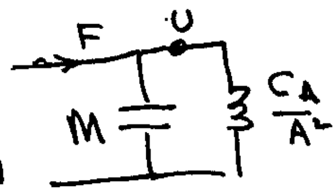




$C_A = \frac{\forall}{\rho_0 c^2}$



$$\left.\begin{array}{l}\frac{F}{A}=P\\ uA=Q\end{array}\right\}\Rightarrow \frac{u}{F}=\frac{1}{A^2}\frac{Q}{P}=\frac{1}{A^2}C_A s$$



$$\frac{U}{F} = \frac{1}{Ms} \parallel \frac{C_A}{A^2}s$$

$$\frac{U}{F}=\frac{C_A s}{MC_A s^2+A^2}=\left(\frac{1}{M}\right)\frac{s}{s^2+\frac{A^2}{MC_A}}$$

poles

$$j\omega = s = j\sqrt{\frac{A^2}{MC_A}}$$

$$\omega = \sqrt{\frac{A^2}{MC_A}}$$

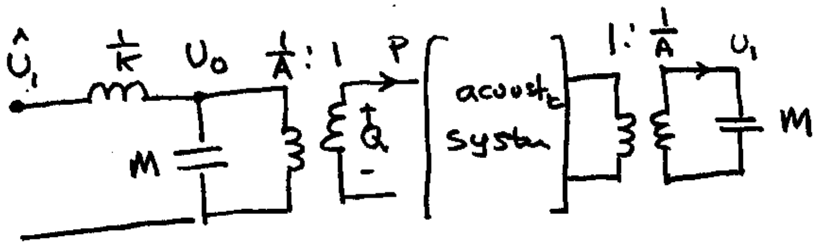
if $f(t) = \delta(t)$

$F(s) = 1$

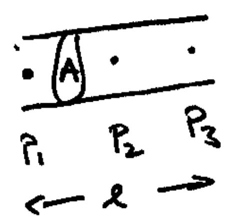
$$\boxed{U(s) = \frac{1}{M}\,\frac{s}{s^2+\frac{A^2}{MC_A}}}$$

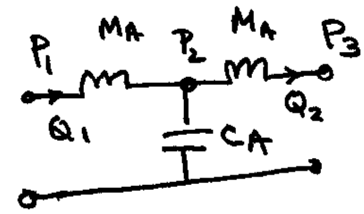
$t>0$

$$u(t) = \frac{1}{M}\cos\left(\sqrt{\frac{A^2}{MC_A}}\,t\right)$$

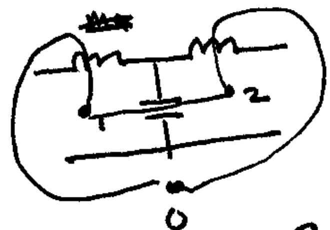


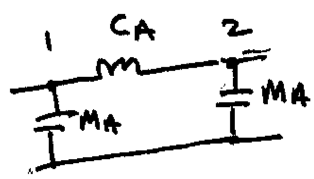
Acoustic system

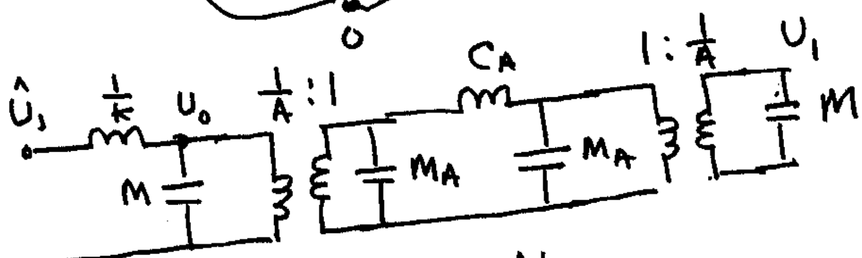




Dual





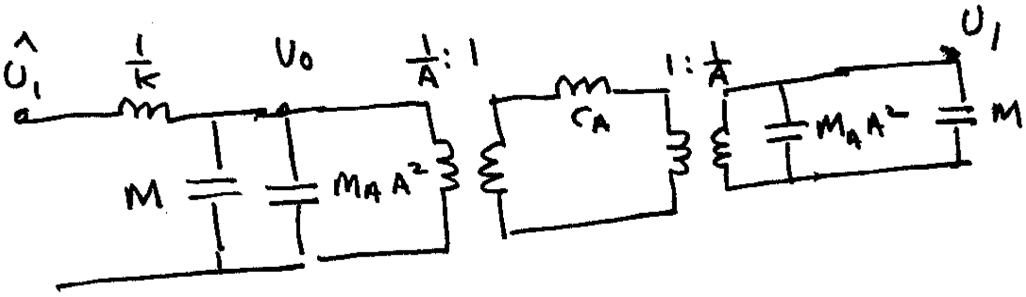


$$C_A = \frac{AL}{\rho_0 c^2}$$

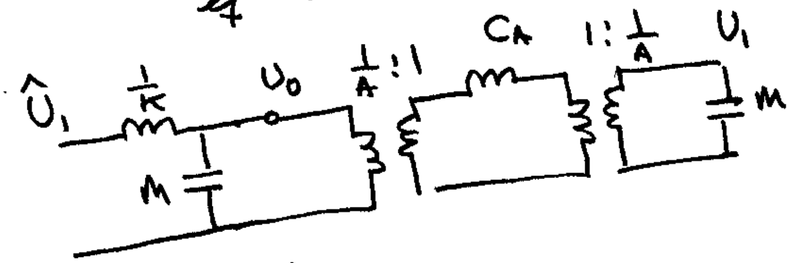
$$M_A = \frac{\rho_0 L/2}{A}$$

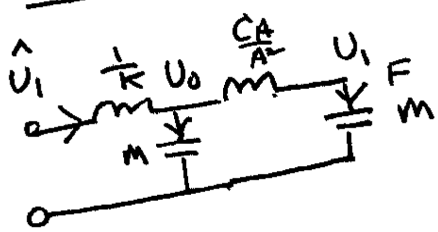
$$M_A << \frac{M}{A^2}$$

push impedance across transformer



if $M >> M_A A^2$





(b) $$\frac{U_1}{U_0} = \frac{\frac{1}{Ms}}{\frac{C_A}{A^2}s + \frac{1}{Ms}}$$

(c) $$F = \frac{U_0}{\frac{C_A}{A^2}s + \frac{1}{Ms}}$$

(d) $$\frac{\hat{U}_1 - U_0}{\frac{s}{k}} = \frac{U_0}{\frac{1}{Ms}} + \frac{U_0 - U_1}{\frac{C_A}{A^2}s}$$

$$\frac{U_0 - U_1}{\frac{C_A}{A^2}s} = \frac{U_1}{\frac{1}{Ms}}$$

$$\left.\begin{aligned} k(\hat{u}_1 - u_0) &= M\ddot{u}_0 + \frac{A^2}{C_A}(u_0 - u_1) \\ \frac{A^2}{C_A}(u_0 - u_1) &= M\ddot{u}_1 \end{aligned}\right\}$$