University of Massachusetts Lowell
Department of Electrical and Computer Engineering
16.413 Linear Feedback

1. Given the state equations

$$\begin{bmatrix} x'_1 \\ x'_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

   a. Determine the eigenvalues and eigenvectors of the system matrix.

   b. Determine the state-transition matrix.

2. Given the state equations

$$\frac{d\underline{x}}{dt} = \begin{bmatrix} 0 & 1 \\ -6 & 5 \end{bmatrix} \underline{x} + \begin{bmatrix} 1 & 1 \\ 6 & 1 \end{bmatrix} \underline{u}(t)$$

$$\underline{y} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \underline{x}$$

   a. Determine the eigenvectors of the system matrix given the eigenvalues are equal to 2 and 3.

   b. Determine if the system is controllable and observable. Justify your answer.

3. Consider the system

$$\frac{dx}{dt} = x + u$$

with performance index

$$\Pi = \int_0^1 u^2 dt$$

   a. Find the state function of Pontryagin H.

   b. Using the state function determine the optimal input u^0.

   c. Determine the governing equations for λ and x.

   d. Find optimal input given $x(0) = 0$ and $x(1) = 1$.