

$$1. (a) \frac{C(s)}{R(s)} = \frac{(s+1)\frac{10}{s^2}}{1 + \frac{10(s+1)}{s^2}}$$

$$\frac{C(s)}{R(s)} = \frac{10(s+1)}{s^2 + 10s + 10}$$

$$(b) \text{ poles: } \boxed{s = -5 \pm \sqrt{15}} \quad \begin{matrix} = -8.9 \\ = -1.1 \end{matrix}$$

step response

$$C(s) = \frac{1}{s} \left\{ \frac{10(s+1)}{\{s+5+\sqrt{15}\}(s+5-\sqrt{15})} \right\}$$

$$c(t) = e^{st} C(s) \Big|_{s=0} + e^{st} C(s) \Big|_{s=-5-\sqrt{15}} + e^{st} C(s) \Big|_{s=-5+\sqrt{15}}$$

$$= 1 + \left[\frac{10}{-5-\sqrt{15}} \right] \frac{(-5-\sqrt{15}+1)}{(-2\sqrt{15})} e^{-(5+\sqrt{15})t} + \left[\frac{10}{-5+\sqrt{15}} \right] \frac{(-5+\sqrt{15}+1)}{(2\sqrt{15})} e^{(-5+\sqrt{15})t}$$

$$= 1 - 1.145 e^{-8.9t} + 0.145 e^{-1.1t}$$

$$c(t) = 1 - 1.145 \left\{ e^{-8.9t} - e^{-1.1t} \right\} \quad t > 0$$

B 4-13

$$\frac{C(s)}{R(s)} = \frac{G \frac{1}{s}}{1 + G \frac{1}{s}} \quad G = \frac{16}{s + 0.8 + 16k}$$

$$= \frac{16}{s(s + 0.8 + 16k) + 16} = \frac{16}{s^2 + (0.8 + 16k)s + 16}$$

$$2\omega_n \zeta = 0.8 + 16k \Rightarrow 16k = 2(4)(0.5) - 0.8 \Rightarrow \boxed{k = 0.2}$$

$$\omega_n^2 = 16 \Rightarrow \boxed{\omega_n = 4}$$

$$\boxed{\zeta = 0.5}$$

$$t_r = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)}{\omega_d} = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{\frac{3}{4}}}{\frac{1}{2}}\right)}{4\sqrt{\frac{3}{4}}} = 1.20 \text{ sec}$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = \frac{\pi}{4\sqrt{\frac{3}{4}}} = 0.906$$

$$M_p = e^{-\frac{\zeta}{\sqrt{1-\zeta^2}} \pi} = e^{-\frac{1}{2} \sqrt{\frac{3}{4}} \pi} = 0.66$$

$$27b: t_s = \frac{4}{\zeta \omega_n} = \frac{4}{4(\frac{1}{2})} = 2$$