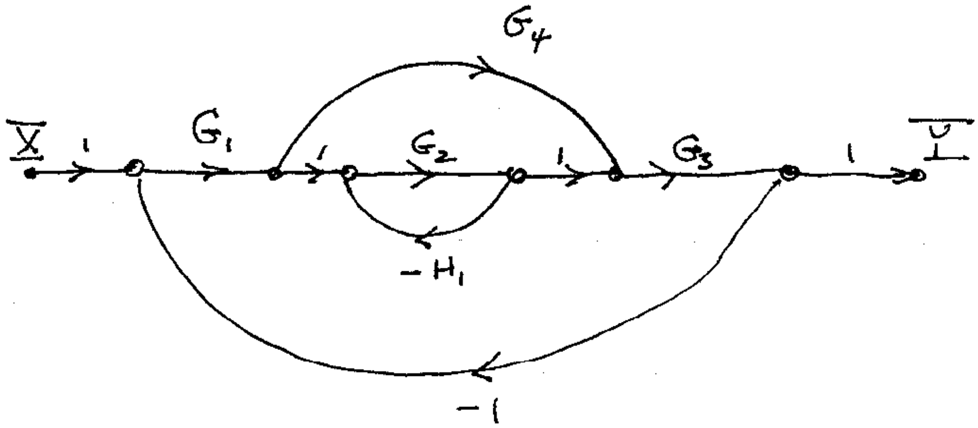
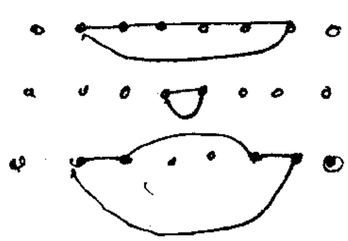
1. 

(a) Forward Path:

$P_1 = G_1 G_2 G_3$

$P_2 = G_1 G_4 G_3$

(b) Loops



$L_1 = (-1) G_1 G_2 G_3$

$L_2 = (-H_1)(G_2)$

$L_3 = (-1) G_1 G_4 G_3$

$\Delta = 1 - (L_1 + L_2 + L_3) + L_2 L_3$

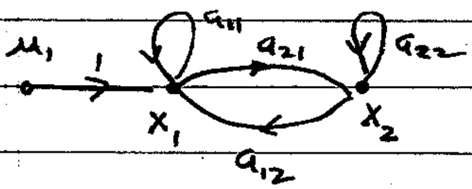
$\Delta_1 = 1$

$\Delta_2 = 1 - L_2$

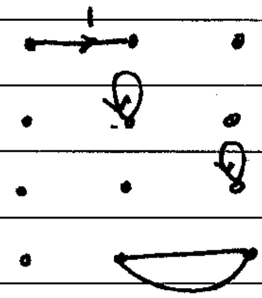
$$\frac{Y}{X} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

2.

$$X_1 = a_{11} X_1 + a_{12} X_2 + u_1$$
$$X_2 = a_{21} X_1 + a_{22} X_2$$



(a) $\dfrac{X_1}{u_1}$ FP: LP:



$P_1 = 1$

$L_1 = a_{11}$

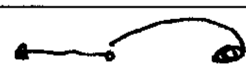
$L_2 = a_{22}$

$L_3 = a_{21} a_{12}$

$$\Delta = 1 - [L_1 + L_2 + L_3] + L_1 L_2 = 1 - a_{11} - a_{22} - a_{21}a_{12} + a_{11}a_{12}$$
$$\Delta_1 = 1 - [L_2] = 1 - a_{22}$$

$$\frac{X_1}{u_1} = \frac{1 - a_{22}}{1 - a_{11} - a_{22} - a_{21}a_{12} + a_{11}a_{22}}$$

$\dfrac{X_2}{u_1}$ FP:



$P_1 = a_{21}$

$\Delta_1 = 1$

$$\frac{\bar{X}_2}{u_1} = \frac{a_{21}}{1 - a_{11} - a_{22} - a_{21}a_{12} + a_{11}a_{22}}$$

(2b)

$$\begin{bmatrix} 1-a_{11} & -a_{12} \\ -a_{21} & 1-a_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ 0 \end{bmatrix}$$

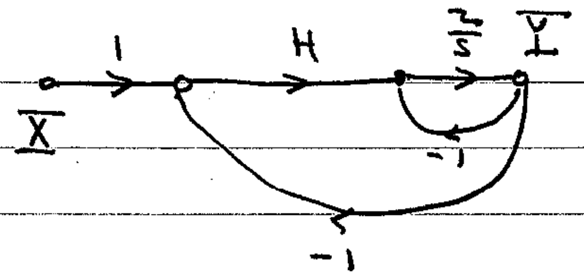
$$X_1 = \frac{\begin{vmatrix} u_1 & -a_{12} \\ 0 & 1-a_{22} \end{vmatrix}}{\begin{vmatrix} 1-a_{11} & -a_{12} \\ -a_{21} & 1-a_{22} \end{vmatrix}} = \frac{u_1\,(1-a_{22})}{(1-a_{11})(1-a_{22}) - a_{12}a_{21}}$$

$$X_2 = \frac{\begin{vmatrix} 1-a_{11} & u_1 \\ -a_{21} & 0 \end{vmatrix}}{\begin{vmatrix} 1-a_{11} & -a_{12} \\ -a_{21} & 1-a_{22} \end{vmatrix}} = \frac{u_1\,a_{21}}{(1-a_{11})(1-a_{22}) - a_{12}a_{21}}$$

$$\frac{X_1}{u_1} = \frac{1-a_{22}}{\Delta}$$

$$\frac{X_2}{u_2} = \frac{a_{21}}{\Delta}$$ ✓

3.



FP: $P_1 = \frac{H2}{s}$

LP: $L_1 = -\frac{H2}{s}$

$L_2 = -\frac{2}{s}$

$\Delta = 1 - L_1 - L_2$

$\Delta_1 = 1$

(a) $$\frac{Y}{X} = \frac{P_1 \Delta_1}{\Delta} = \frac{\frac{2H}{s}}{1 + \frac{2}{s}(H+1)} = \frac{2H}{(s+2) + 2H} = \frac{\frac{2H}{s+2}}{1 + \frac{2H}{s+2}}$$

(b) $$E = X\left[1 - \frac{Y}{X}\right] = X\left[\frac{1}{1 + \frac{2H}{s+2}}\right]$$

$$x(t) = t\,u(t) \Rightarrow X(s) = \frac{1}{s^2}$$

$$e(\infty) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{s}{s^2}\left[\frac{1}{1 + \frac{2H}{s+2}}\right] = \frac{1}{2}$$

$$= \lim_{s \to 0}\left[\frac{1}{s + \frac{2sH}{s+2}}\right] = \lim_{s \to 0} \frac{1}{sH} = \frac{1}{2}$$

$$\lim_{s \to 0} Hs = 2 \quad \therefore \quad H(s) = \frac{K}{s}$$

$$\lim_{s \to 0} \frac{K}{s}s = 2 \qquad \boxed{K = 2}$$

$$\boxed{H(s) = \frac{2}{s}}$$